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ABSTRACT

This is the Teacher's Commentary for Mathematics for the Elementary School, Book 1 (Part 1), Special Edition. The writers have relied on the existing SMSG kindergarten and first grade materials as a framework. This special edition is designed to meet the needs of disadvantaged children. Included in the Commentary are background information to the teacher, discussion of activities in the text, and answers to activities and exercises. (RH)

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SCHOOL
MATHEMATICS
STUDY GROUP

MATHEMATICS FOR THE ELEMENTARY SCHOOL

BOOK 1 (Part 1)

Teacher's Commentary

SPECIAL EDITION (Revised)

U.S. DEPARTMENT OF HEALTH
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MATHEMATICS FOR THE ELEMENTARY SCHOOL

BOOK 1 (Part 1)

Teacher's Commentary

SPECIAL EDITION (Revised)

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PREFACE

The increasing contribution of mathematics to the culture of the modern world, as well as its importance as a vital part of scientific and humanistic education, has made it essential that the mathematics in our schools be both well selected and well taught--at all levels, from the kindergarten through the graduate school.

With this in mind, mathematical organizations in the United States cooperated in the formation of the School Mathematics Study Group (SMSG). The general objective of SMSG is the improvement of the teaching of mathematics in grades K - 12 in the schools of this country. The National Science Foundation has provided substantial funds for the support of this endeavor.

One of the prerequisites for the improvement of the teaching of mathematics in our schools is an improved curriculum--one which takes account of the increasing use of mathematics in science and technology and in other areas of knowledge, and at the same time one which reflects recent advances in mathematics itself. Among the projects undertaken by SMSG was that of enlisting a group of outstanding mathematicians, educators, and mathematics teachers to prepare a series of sample textbooks which would illustrate such an improved curriculum.

The development of mathematical ideas among young children must be grounded in appropriate experience with things from the physical world and the environment. The materials in this publication provide for young children an introduction to the study of mathematics that reflects clearly this point of view, in which growth is from the concrete to the abstract, from the specific to the general. Major emphasis is given to the exploration and progressive refinement of ideas associated with both number and space.

The writers have relied on the existing SMSG kindergarten and first grade materials as a framework. However, the writers hope that this special edition will better meet the needs of disadvantaged children.

It is not intended that this book be regarded as the only definitive way of introducing good mathematics to culturally deprived children at this level. Instead, it should be thought of as a sample of the kind of improved curriculum that is needed and as a source of suggestions for the authors of commercial textbooks. It is sincerely hoped that this and other texts prepared by SMSG will lead the way toward inspiring a more meaningful teaching of Mathematics, the Queen and Servant of the Sciences.

Teachers' Commentary

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Chapter 1

SETS AND NUMBERS

BACKGROUND

A set is just a collection of things. The children in your classroom, the books on the bookshelf, the letters of the alphabet, the numbers used in counting are each a set of things.

The things or objects in the set are called members or elements of the set. The members of a set belong to it. Suppose we consider the set of men who have been president of the United States of America. George Washington is a member of this set. Alexander Hamilton is not a member of this set.

A set may consist of a variety of objects. A prime characteristic of a set is that there is a method or rule whereby set membership or nonmembership can be determined. Suppose we consider the set of objects on the teacher's desk. The criterion for determining whether or not a particular object is a member of the set is, "Is this object on the teacher's desk?"

A set may have many members; it may have a single member, it may have no members at all. If a set has no members it is named the empty set. There is just one empty set although it may have many descriptions. Some descriptions of the empty set are:

the oceans bordering the state of Missouri
the squares with five sides
the counting numbers between 8 and 9
the men students enrolled at Vassar College.

Let us consider a set whose elements are

wagon, bicycle, scooter, doll, block.

A set whose elements are

wagon, bicycle, scooter

is a subset of the original set.
A set whose element is

doll

is a subset of the original set. This set has a single member, the doll.

The study of sets prepare for the concept of number. To understand how we do this let us consider a related problem. How do we convey the notion of red? Suppose a child is beginning to learn about color, and we want him to learn to identify the color red. We would surely point to, or touch, all of the red objects around, trying to lead him to conceive of the color red as the property which is shared by all of these objects.

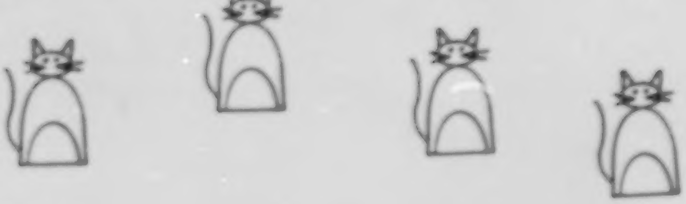
How do we develop a concept of the number seven? It is surely the property which is shared by all of the sets having seven members. But this raises a difficulty. Can we talk about all of the sets having a certain number of members without having the notion of number? Even simpler: can we decide when two sets have the same number of members without knowing how to count them? Can you discover whether there are the same number of boys and girls in a room without counting? Is there a boy for each girl and a girl for each boy? We can pair each boy with a girl. Can we find out whether there the same number of cups and saucers on a shelf without counting? Is there a cup for each saucer and a saucer for each cup? We can pair each cup with a saucer. Thus, *there is a way of deciding whether or not two sets have the same number of members without knowing anything about number*.

Suppose we are given two sets which we wish to compare. We may *pair* each member of the first set with a member of the second set as long as possible. If we run out of members of the first set but not of the second set, we know that there are *fewer members in the first set than in the second*;if both sets are exhausted at the same time we say the first has as many members as the second; and that the second has as many members as the first, or that the sets are *equivalent*.* If the second set is used up first we say there are *more members in the first set than in the second*. We can describe these possibilities by saying we try to set up a *one-to-one correspondence* between the elements of the two sets. If we succeed, the sets are equivalent.

**Equivalent* is *not* synonymous with *equal*. Two sets are *equal* if and only if they have the same members; i.e., the sets A and B are equal if and only if every member of A is a member of B *and* every member of B is a member of A. For example: the set consisting of the President, Vice-President, and the Secretary of State is equivalent to the set consisting of the British Prime Minister, the Chancellor of Exchequer, and the Minister of War, but these sets are certainly not equal.

Suppose we are given two sets which we wish to compare. For example, if we want to compare the set of boys in the classroom with the set of girls, we can form couples as long as possible. If we run out of boys first, there are fewer boys than girls; if we run out of girls first, there are more boys than girls; if everybody has a partner, then there are as many boys as girls, and the set of boys is equivalent to the set of girls. Notice that counting is not necessary.

Manipulations with sets of physical objects are intended to develop an understanding of whole numbers. The number of members of a set, or just the number of a set, is the property common to all sets equivalent to that set. The number four is the property common to all sets equivalent to



or to

We see from the pictures above that the set of cats is equivalent to the set whose members are a ball, a cap, a chair, and a box - it is easy to set up a one-to-one correspondence between these sets. The number of one set is the same as or equal to the number of the other set.

Thus, the notion of equivalence underlies the concept of whole numbers. We will see that the notions of "more than" and "less than" lead us to the idea of inequality of number.

Frequently, the elements of a set present themselves in a natural order. For instance most English speaking people would list the members of the set of vowels as a, e, i, o, u. It is natural to list the elements in this order because this is the order on which they were learned. It is convenient because without undue checking one can be sure he has not omitted any member. Similarly, it is natural to list the members of the set of letters of the alphabet as

a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z.

In ordinary writing we write this set as

a, b, c, ..., z.

The three dots, ..., mean "and so on in the same manner". They are used to indicate the omission of certain members.

Essentially, to "order" things is to list or arrange them in some particular fashion. One can then say of each element which of the other elements it "precedes". We do this by comparing pairs of elements in the list and deciding which element precedes the other. The word "precedes" may be replaced by "above", "below", "shorter than", "greater than" and so on, depending on the elements to be ordered.

For example, consider the set of names

James, Wilson, Smith, Alton

If we order these elements alphabetically we have

Alton, James, Smith, Wilson.

We call this set an *ordered set*.

Let us establish some ordered sets beginning with the set whose element is 1. We continue

1, 2,
1, 2, 3,
1, 2, 3, 4,

and so on.

We see that each of these sets is a subset of each of the following sets. Thus,

> the set whose element is 1 is a subset of the set whose element is 1, 2;
>
> the set whose elements are 1, 2, is a subset of the set whose elements are 1, 2, 3, and so on.

By comparing these sets, called *standard sets*, we can determine which belongs before the others in ordering these sets. For example, we see immediately that the set whose members are 1, 2, 3 belongs before the set whose members are 1, 2, 3, 4, 5, 6 in ordering these standard sets.

Now let us consider the sets with the following elements:

Joe, Bob;
✱, ○
Doris, Nora;
Bill, Fred.

Each of the sets is equivalent to any other in this list. Let us consider all the sets equivalent to any one of these given sets, for example, the set whose members are Bill, Fred. Among the sets equivalent to this set is the standard set whose elements are 1, 2. These sets all possess a common property; their equivalence to the standard set with elements 1, 2. This property is independent of the elements of the set. We call this common property the number two. We say the number property or the number of the set Bill, Fred is 2.

To each set of equivalent sets, we assign a spoken name and written symbols--words and numerals--to name the common property, the number.

The ordering of sets shows us how to order numbers. We say that 5 is *greater than* 3, and that 3 is *less than* 5 because, if the members of a set of 3 are paired off with the members of a set of 5, there will be members of the latter set left over. Notice that if one set has *more* members than another, then the number of the first set is *greater* than the number of the second set; if the first set has *fewer* members, then the number of the first set is *less* than that of the second; and if sets are equivalent then they have the *same* number. The words "more than," "fewer than," and "equivalent" refer to sets. The corresponding words for number are "greater than," "less than," and "equals."

The number of a set may be found by counting the members of the set. At this stage, the children will learn to find the number property of sets with at most ten elements with relatively few errors. For sets with fewer than 6 members, we hope for an identification of the number property without counting. We try to build this perceptive ability by many examples of such sets arranged in different sorts of patterns.

There are two important facts which children should discover in connection with counting. First, the order in which a set is counted is immaterial. Another child counting the same set but starting with a different member and proceeding in a different order will get the same number. Second, if two sets are equivalent, counting will yield the same result. This last fact shows us that the name we assign by counting a set really depends only on the *number* of the set.

1-1. SET AND MEMBER

OBJECTIVE: To introduce the ideas of *set* and *member of set* and the use of these terms to describe things we observe around us.

VOCABULARY: Set, member.

MATERIALS:
1. A variety of small objects: books, blocks, pencils, crayons, sheets of paper, paint brushes, scissors, game boxes, beads, pegs, balls, clothes pins, bottle caps.
2. A variety of flannel board objects: animals, fruit, stars, trees, story-book characters, stick figures, etc. (Exclude geometric shapes at this time.)
3. Magazine pictures of collections: a family, automobiles, telephones, clothing, food, toys, planes, trains, and any other appropriate illustration.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Use the terms "set" and "member" informally but systematically with the children before any formal class work on sets.

For example:

This is our set of books.

Is this your set of blocks? Is this red block a member of your set of blocks?

Is John a member of the set of boys in the classroom?

Children may be familiar with sets of dishes, silverware, blocks, etc. If not, use sets of things that are familiar to children in your class. Such examples are helpful when first developing the meaning of the term *set*. The following may be used as a guide.

Ask several children to name the members of each of the following:

The set of things he would like for Christmas;
The set of children seated at his table;
The set of things he likes to play with;
The set of colored crayons he'd like to use.

In each case, mention specifically the members of the set. For example, refer to Jim's set of Christmas presents as bicycle, baseball, airplane or to Joe's set of friends at his table as Mary, Bill, Leroy, etc. Since the members of a set need not show any relationship other than that they are members of the same set, it is helpful to use such sets early in the program. For example, place a set of flannel cut-outs such as a bird, apple, star, and rabbit on the flannel board. After the children have named the members of the set, ask questions such as:

> *Is the rabbit a member of this set?* (Yes.)
> *How do you know this is true?* (We said the rabbit was a member of the set. There is a flannel cut-out of a rabbit on the flannel board. It's there.)
> *Is a dog a member of this set?* (No.)
> *How do you know the dog isn't a member of this set?* (The dog doesn't belong to the set. There isn't a cut-out of a dog on the flannel board. The dog isn't there.)

Similar questions may be asked about other things which may or may not be members of the given set. Then show the children other collections and ask them to name the members. Children can be asked to pick a collection of objects or cut-outs and request the class to name the members. Refer to each set as a set as well as a collection. These descriptions of sets should be explicit.

> *We have many books in our room. How can we talk about these books?* (A set or collection of books in our room.)
> *Let's look at this set of blocks* (none of the blocks should be the same color) *on my desk.*
> *What blocks are members of this collection?* (a blue block, a red block, etc.)

Use sets which have only a single member.

> *We have a set of teachers in this room. Who are the members of this set?* (Miss or Mrs. ____)
> *Are there any other members of this set?* (No.)
> *Look at the set I have placed on the flannel board* (a single star, rabbit or apple).
> *Is this a set?* (Yes.)
> *Name the members of this set.* (A star.)

If the children use the term *one* star, say:

> *Yes, that is true. There is a single member in the set of stars on the flannel board.*

The ideas of set and member should be used throughout the day in classroom activities if the usage is not overly contrived. Correct language on the part of the teacher will eventually make these terms a part of the children's vocabulary . No attempt should be made to drill on these terms. If the children's responses indicate that they understand the meaning of the terms, the teacher should show approval in some way and repeat the correct terminology.

USING THE PUPIL'S BOOK, pages 1-6.

These are teaching pages. Be sure that each child has his book opened to the correct page. Display the page so that all children can see it. Read the title and point to the words as you read.

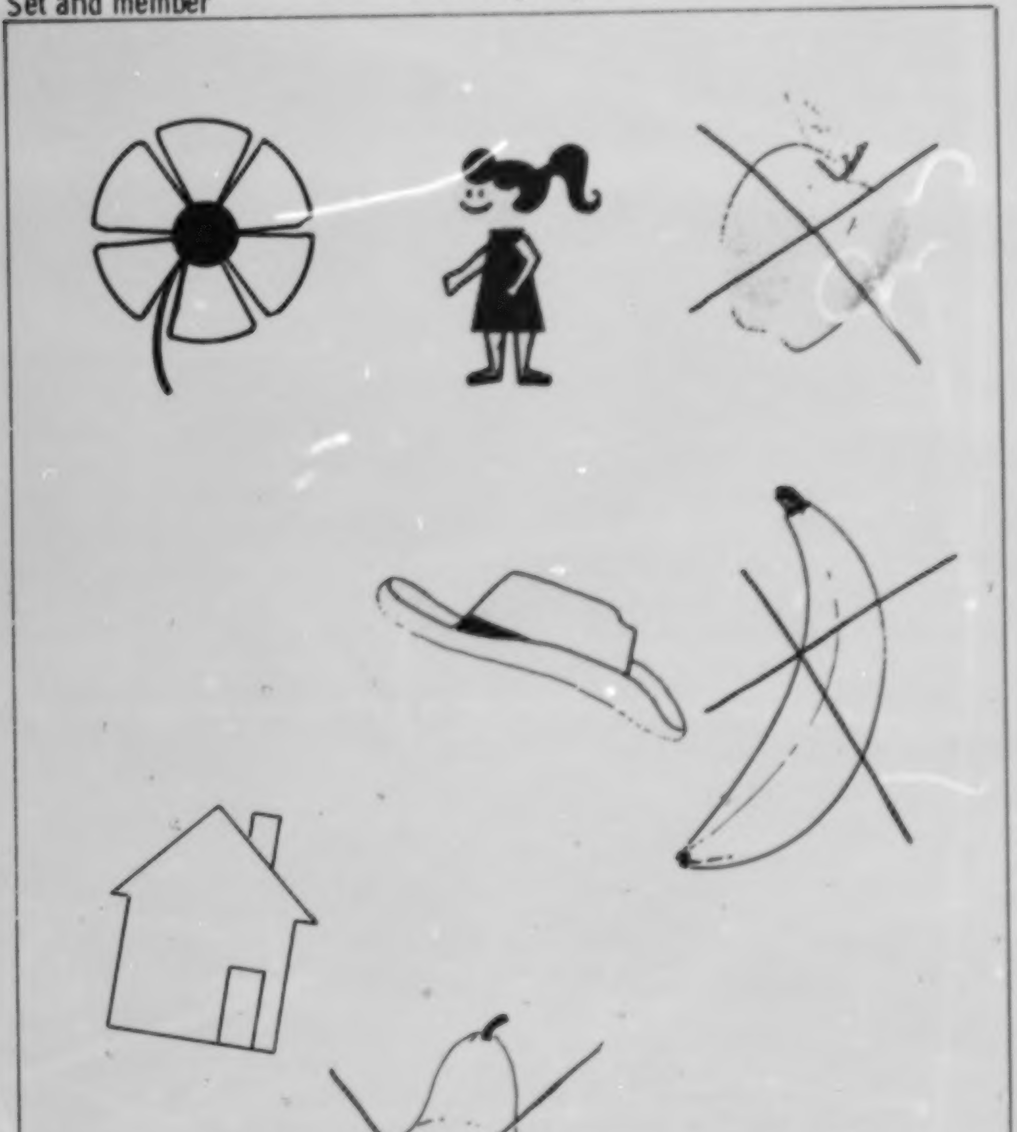
Select a child to come to the front and name the members of the set shown on the page. Be sure that he points to each member as he says the name of the member.

Give the child a crayon and on the pages listed below, ask him to mark with an X the members of the set indicated:

Page 1: the set of fruit. (You may have to prepare the children for the children for this lesson by talking about various kinds of "fruit". The word "fruit" may be unfamiliar to some children.)
Page 2: the set of animals.
Page 3: the set of things to eat.
Page 4: the set of toys with wheels.
Page 5: the set of things to wear.
Page 6: the set of things that fly.

Teaching Page

Set and member



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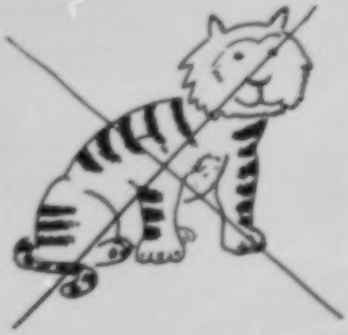

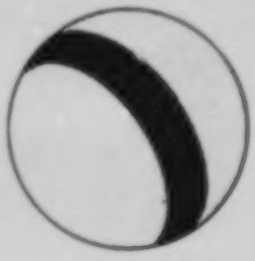

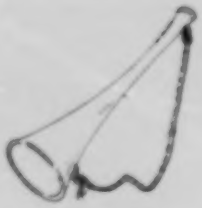

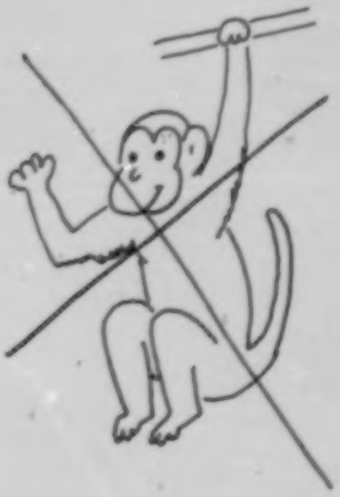

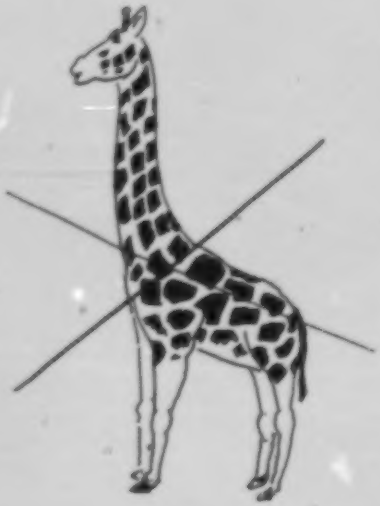

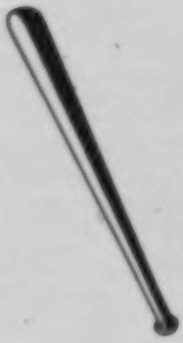

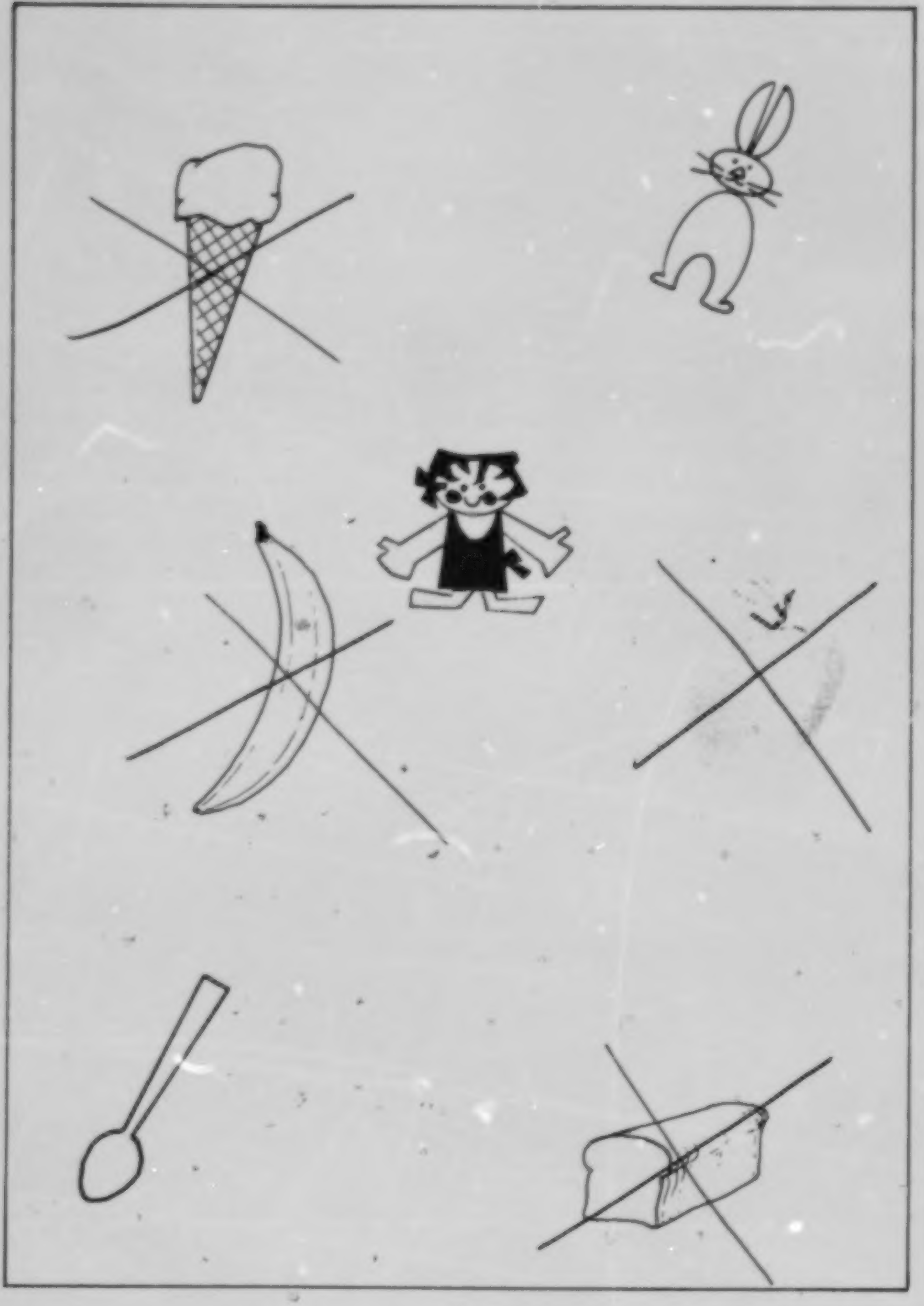

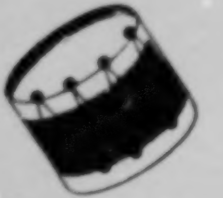

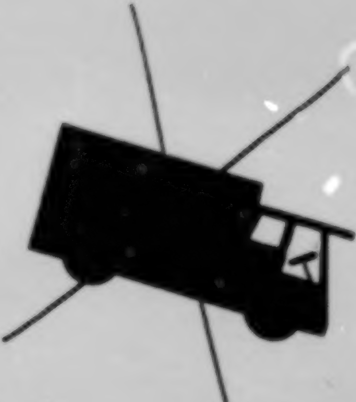

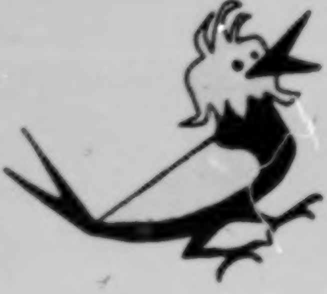

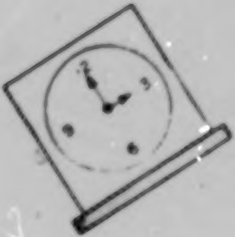

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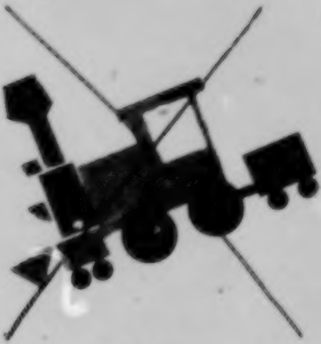
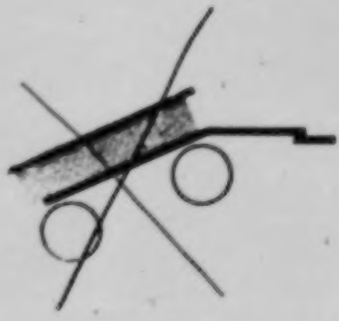
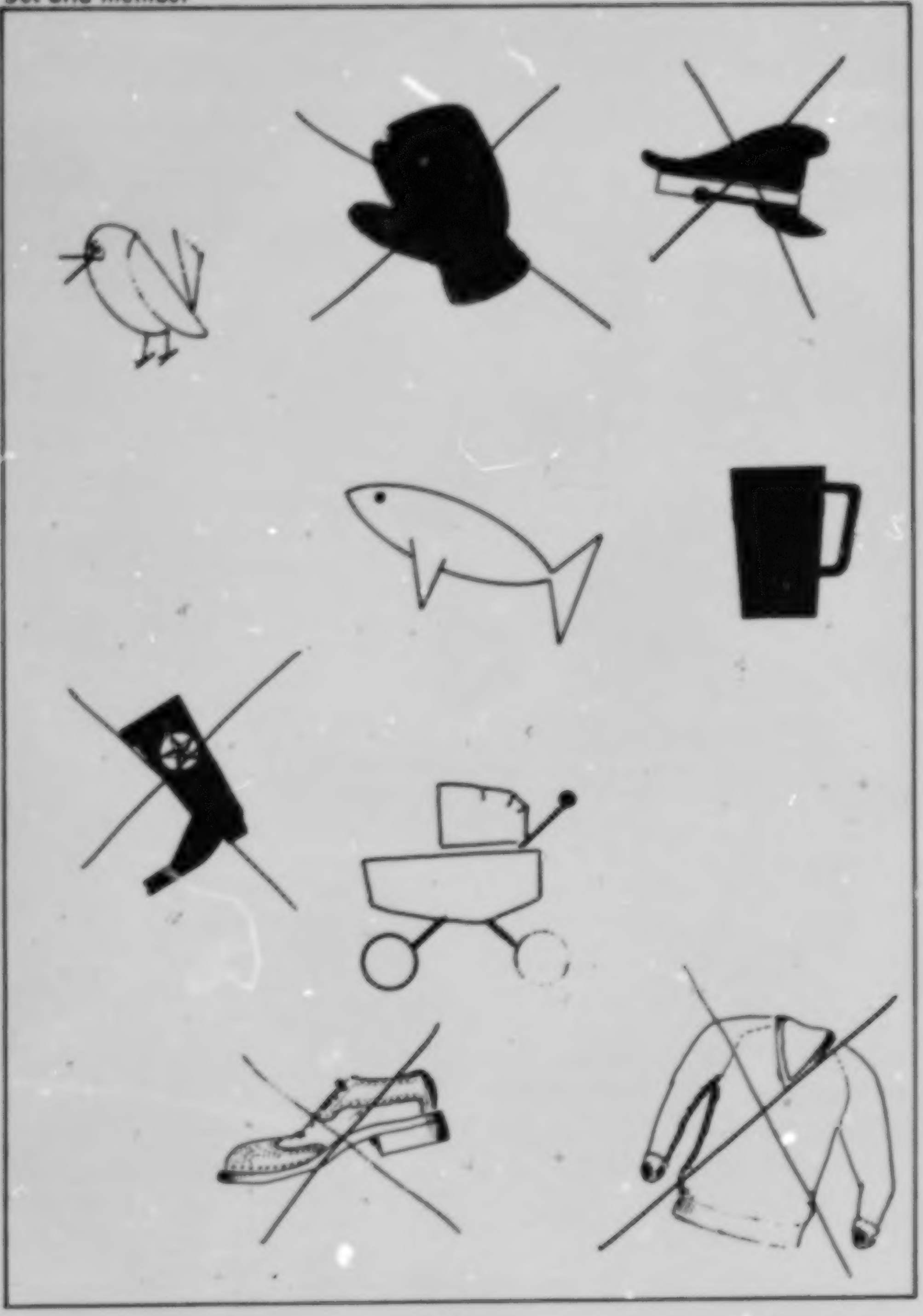


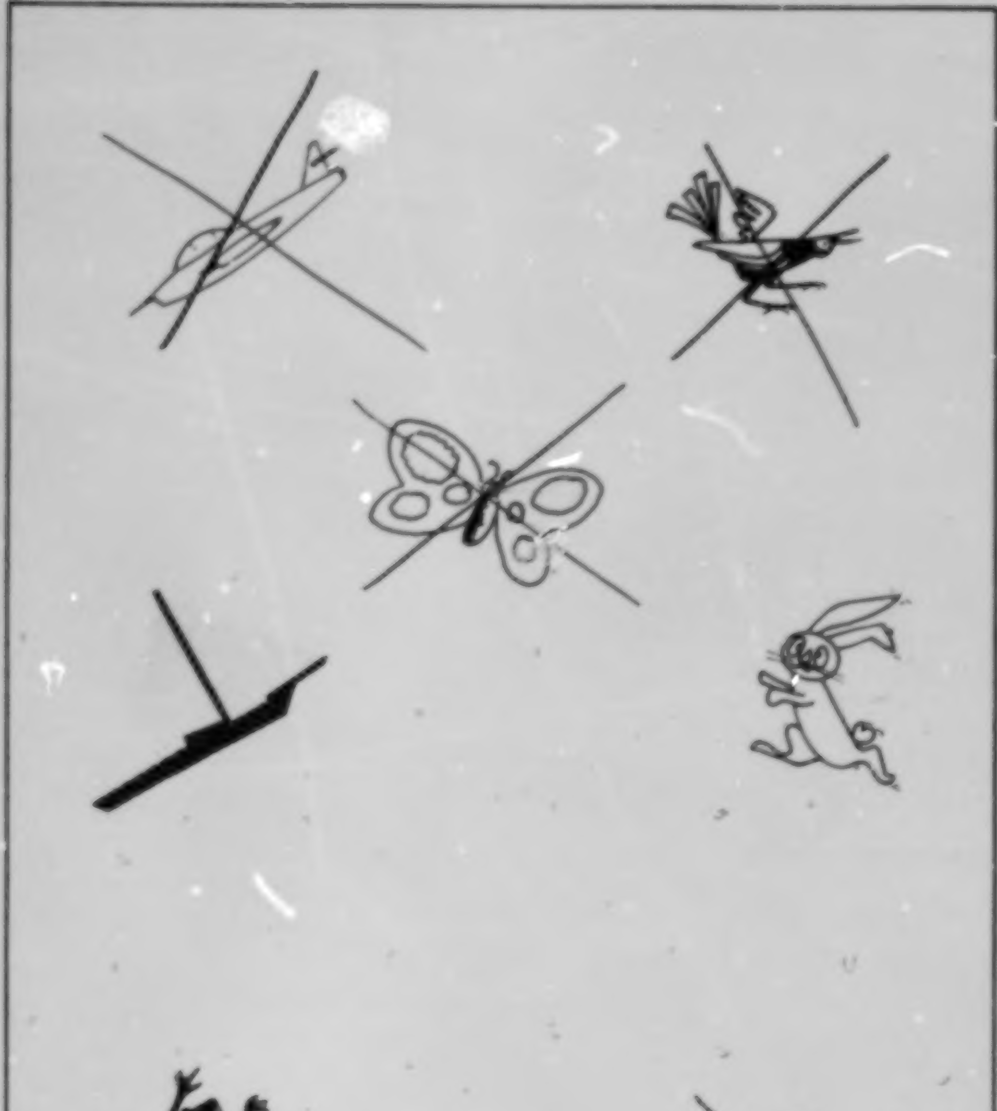


Teaching Page

Set and member



Set and member



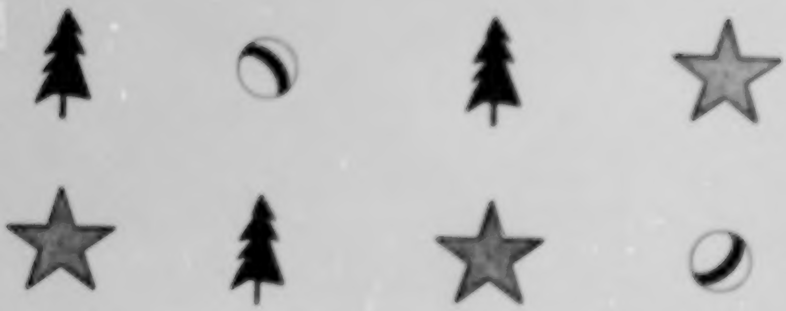
FURTHER ACTIVITIES:

1. Art activity: Have children draw pictures of sets and orally name their members. Suggestions: child's own family, story-book family, toys, things that grow in the yard.
2. Oral reading of a story dealing with a family or set of some sort.
3. Provide old magazines and have children cut out pictures of sets. These may be mounted on art paper.
4. The daily calendar may be used in conjunction with the vocabulary introduced. (The set of days in the week, the set of days we go to school.)
5. Four or five children are assigned a shoe-box or cigar box as "homework". They are to bring a set which can be used for the classwork the following day. Some discussion may be needed concerning size, quantity, etc. of items which could be brought for this purpose. This activity is helpful in providing a variety of materials used and serves in a small way to familiarize parents with the child's work.

The following lesson is suggested in conjunction with "the family" presented in most children's readers. Use illustrations and make references to this family to clarify the discussion.

> *We read a story about a family today.*
> *The family is a collection of people.* *Is that a set?* (Yes.)
> *Who were the members of this family?* (Use flannel board material or pictures to suggest mother, father, etc.)
> *Each one in the family is an important member of it.*
> *Just as each person in the family is a member of that set, so is each thing in any set called a member* (e.g., children are members of our class.)
> *Let's think of some of the sets we found in our room and name their members.* (Review the sets previously discussed and decide what their members are. First describe the set; then name its members.)

Point out that some things are *not* members of some sets. For instance: The books are not members of the set of paint brushes; Johnny is not a member of the set of girls. Children enjoy answering such questions as: "Is the flag a member of the set of books?" Be sure to emphasize that the members of a set need not be the same kind of objects. You might use a flannel board illustration of the following sort.



Be sure that the classroom activities include considerable manipulation by the children of sets of objects. Worksheets may be used (you might ask the children to draw a ring around the set of stars, or the set of trees, on a worksheet) but physical manipulation of objects is essential. Key relationships that children need to understand are extremely difficult to convey in pictures because the pictures do not permit movement of the objects shown. Furthermore, the actions of the children contribute to learning.

1-2. THE EMPTY SET

OBJECTIVE: To develop the idea of the set with no members.

VOCABULARY: Empty set.

BACKGROUND NOTE:

Zero is the number of members of the empty set.

MATERIALS:
1. Small objects: disks, paper clips, keys or like material.
2. Four containers (covered boxes, bags) in which to place collections of materials.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

A concept of the set with no members can be developed through an activity such as the following:

Place collections of materials in three of the four boxes. Begin the lesson by observing the four boxes and saying that we are looking for a special set in one of the boxes. Have one child at a time select the box that he thinks is the one that they are trying to identify. Ask him to open the box and describe the set of objects in the box and also to name the members of the set. (Sets may consist of members which are very dissimilar, they may be dissimilar only in color, e.g., red, yellow, and blue pencils, or the members may be similar.) One of the boxes should contain a set with a single member, and the "special" box should contain a set which has no members. As the child selects this particular box, he will note that the box is empty.

This "special" box is empty.
What do you think the name of this set is? (The empty set.)
Yes, when there are no members in a set, it is called the empty set.
Bill has some pieces of candy on his desk and nothing else.
He ate all the candy.
What set of things does he have on his desk? (The empty set.)

You may wish to use children's pockets instead of boxes or bags, to repeat the kind of activity suggested above and again observe sets with a single member and the set with no members.

USING THE PUPIL'S BOOK, pages 7-9:

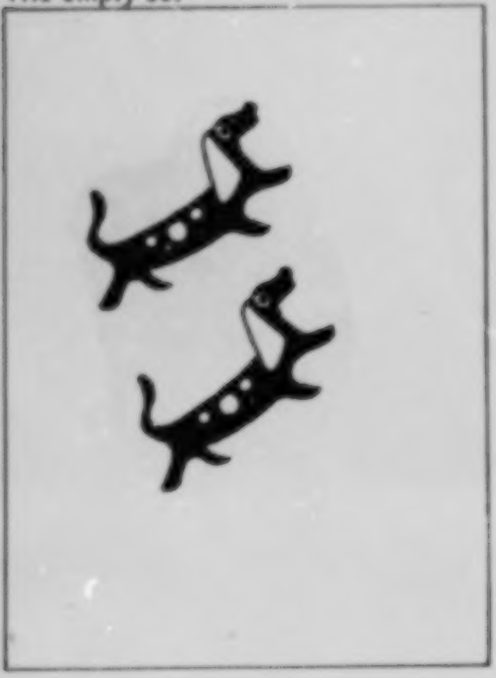
Page 7: is a teaching page. Pages 8 and 9 are for the children to do independently.

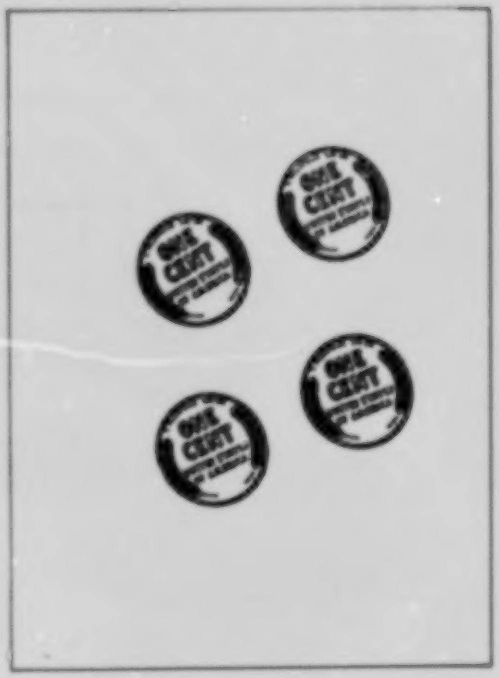
In using Page 7, have the children open their books to the correct page. Display your books so that the children can see the page. Read the title, pointing to the words as you read. Point to one of the sets. Call on a child to name the members of this set. Do the same with the other sets. When the children are asked to name the members of the empty set, be sure that they recognize the fact that it has no members. Say, "This set has no members. What do we call the set that has no members?" (The empty set.) Select a child to come to the book which is displayed. Give him a crayon and instruct him to mark the empty set with an X. Tell the other children to do the same in their books.

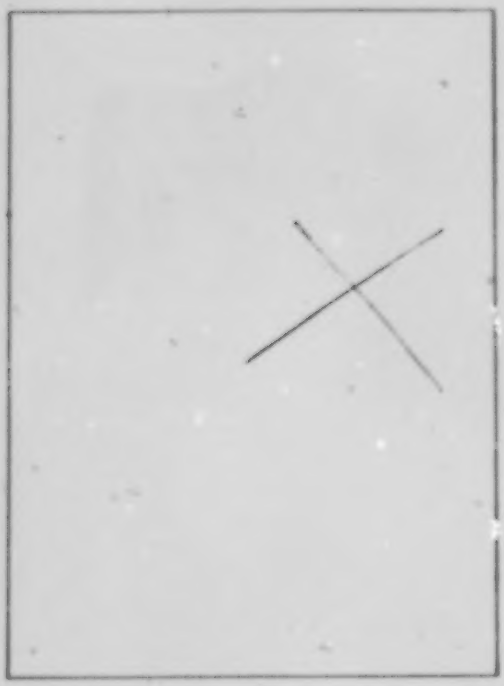
Direct the childrens attention to Pages 8 and 9. Give them instructions as to what to do. They are to mark the empty set with an X. Give help where needed.

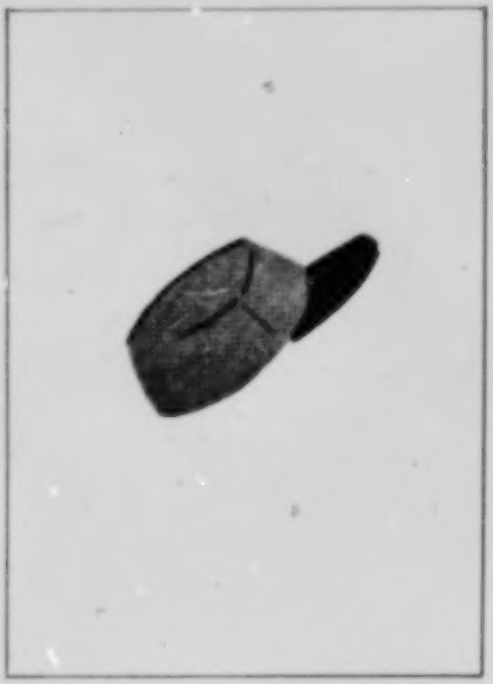
Teaching Page

The empty set

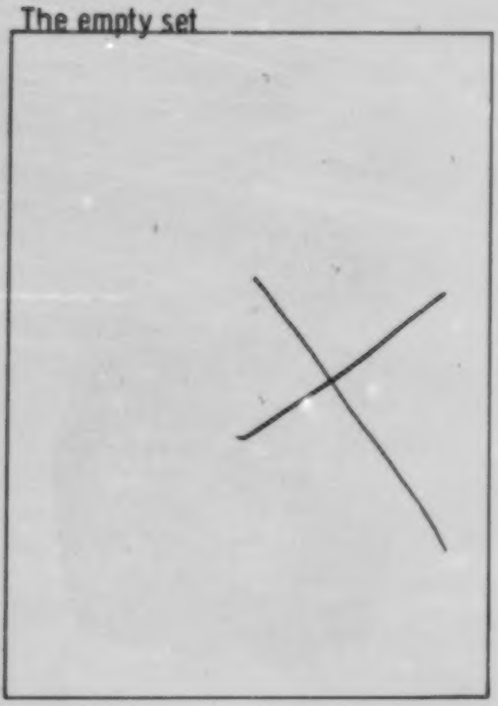


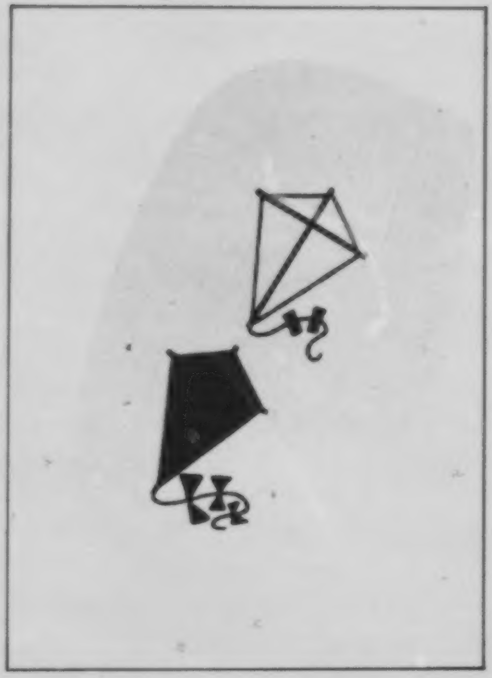

ONE CENT
ONE CENT
ONE CENT
ONE CENT

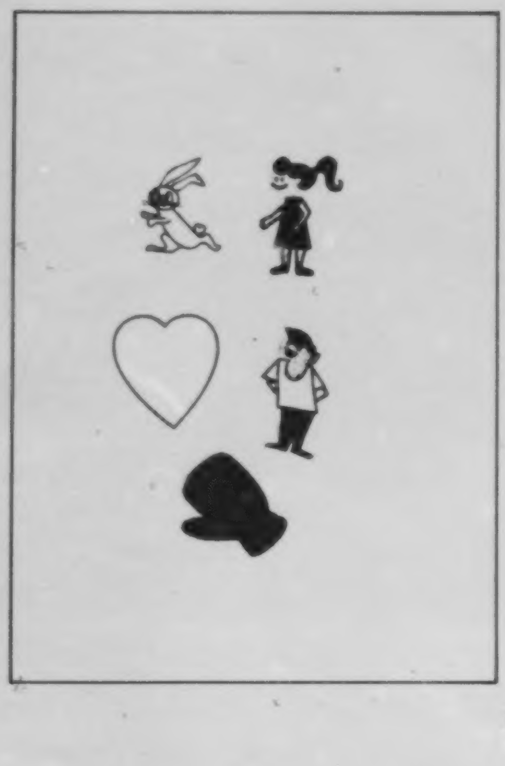


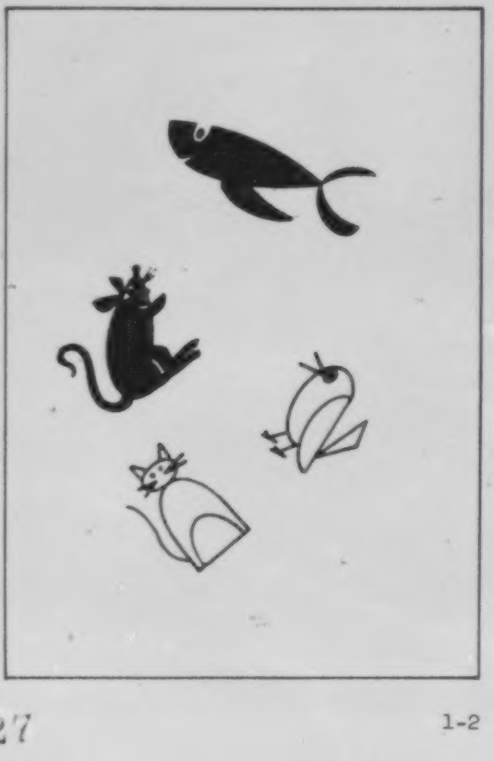


The empty set

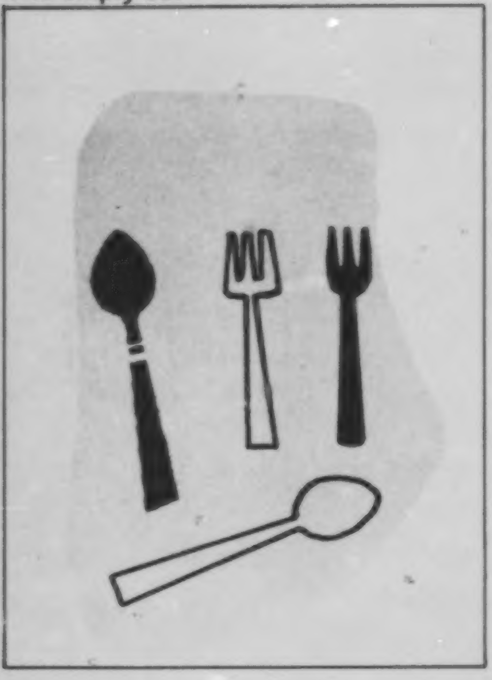


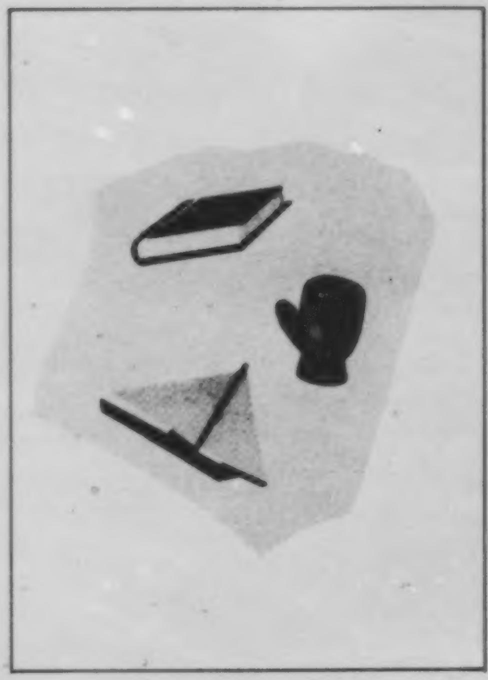


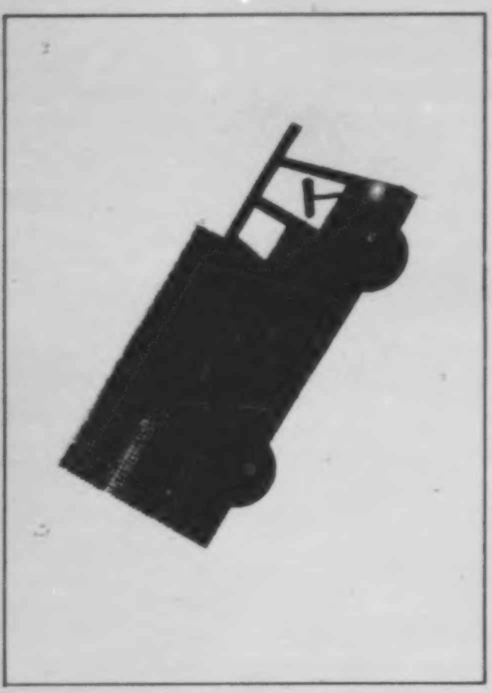


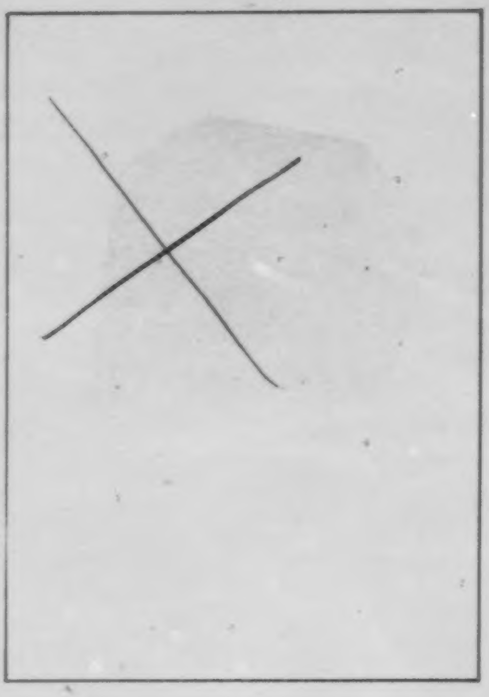


The empty set









1-3. PAIRING AND EQUIVALENCE

OBJECTIVES: To introduce the idea of pairing members of sets and the idea of equivalence of sets.

VOCABULARY: Pair, equivalent, as many as.

BACKGROUND NOTE:

Two sets are equivalent if we can pair each member of the first set with just one member of the second set in such a way that every member of the second set is used. The notion of equivalent sets is fundamental in building number concepts, and an important fact about counting is that if two sets have the same number of members then they are *equivalent*, and if two sets are equivalent they have the same number of members.

MATERIALS: 9" x 12" paper, crayons, name cards, materials for the flannel board, small objects for individual use.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

The following lesson concerns pairing the members of a set of children with the members of a set of name cards. You may prefer to use a set of boys and a set of girls, or a set of cups and a set of saucers, or a set of children and a set of chairs, or some other one of the many pairing situations which occur in the classroom.

Have name cards for only those children present so that sets of cards and children are equivalent. Say:

> *I had a set of name cards for the children in our class.*
> *I have removed the set of cards of the children who are absent.*
> *What cards do I have now?* (The set of name cards of the children here.)

Ask the children to suggest ways that the cards might be passed out. Say:

> *As I hold up your card, please come, get the card, and put it on your desk. Are there as many name cards as there are children?* (Yes.)
> *How do you know that is true?* (Every child has a name card. There aren't any name cards left over.)
> *Yes, there are as many name cards as there are children.*
> *We have PAIRED each member of the set of name cards with a member of the set of children.*

At the top of the flannel board, put a set of objects. This set might consist of an apple, a star, a rabbit and a bird; also display another set consisting of two apples and two stars. Have the sets described.

> Are there as many members in the first set as in the second set? (Yes.)
> How can we be sure?

If a child suggests that there are 4 objects in each set, say

> Yes, that is true. How could we be sure if we didn't know how to count? (Pair.)
> Since we don't want to remove the objects from the flannel board, we can use this yarn to show the pairing.
> Watch as I do this.

After the members of the sets are paired, remove the yarn and ask a child to start the pairing using a member of each set.

> Use this yarn to show the pairing. Choose any cut-outs you wish.

Continue this process until all the members of the sets are paired. Stress the fact that it makes no difference which member of a set you choose to pair with a member of another set.

> Does the pairing show that one set has AS MANY members AS the other. (Yes.)
> When members of one set are paired with members of another set and there are no members left over, we say that the sets MATCH.
> Do these sets match? (Yes.)
> How did we find out if one set has AS MANY members AS the other or that they matched? (Paired.)

Repeat this procedure, using other sets of flannel cut-outs to make certain that the meaning of pairing and matched is understood.

*Provide each child with six or seven set objects on his desk. You will need an equal number of objects for the flannel board.

> Let's see if you can pair the members of a set on your desk with the members of a set I put on the flannel board.

Then you should place one object (rabbit) on the flannel board. Ask the children to put a set that matches in the middle of their desks so that you can quickly see them.

---

*This notation is used to denote a variation in either the activity used or the idea being developed.

I will put another object with my set.
Now I have a new set.
Does your set match my new set? (No.)
What will you need to do to your set?

After the children have placed another object in the middle of their desks ask,

Do our sets match? (Yes.)
How can we be sure? (Pair.)
Mary, will you bring your set up to the flannel board and show us that the sets match?

After making several such verifications, introduce the term equivalent as another word for match.

When sets match, we say that they are equivalent.
Is your set equivalent to mine? (Yes.)
Does your set have as many members as mine? (Yes.)
How did we show that your sets are equivalent to mine? (Paired.)

After clearing the flannel board and the desks repeat the same procedure starting with a set of 3 objects and then a set of 2 objects. Remove all objects from the flannel board and the desks. Start again with a set of 3 objects. After the children have constructed an equivalent set, say, of buttons, remove a member of your set and continue as before. Repeat, using other sets of cut-outs with no more than 4 or 5 members so that the children can see without counting. Make certain that the children can make the distinction between sets that match or are equivalent and sets that do not match and are not equivalent.

Since the terms "as many as" and "equivalent" are difficult for some children to pronounce, accept the term "match" as a suitable substitute and reinforce the other terminology by such comments as:

Yes, these sets match or are equivalent.
If sets are equivalent, it means that each set has as many members as the other.
Your are correct. These sets do not match.
They are not equivalent.
Each set does not have as many members as the other.

Do not attempt to make each child use the terminology; do not drill on these terms. Consistent usage by the teacher will develop this vocabulary and the children will use these words as they gain confidence in their ability to pronounce them correctly.

Some experience in drawing lines to show the pairing of the members of two sets before using the pages in the book is desirable. This may be done by representing sets of objects on the chalkboard and asking children what they might do to show the pairing. Since the use of yarn or flannel pieces is obviously inappropriate they will suggest making lines to show the pairing. Provide an opportunity for all of the children to have this experience at least once.

USING THE PUPILS'S BOOK, pages 10-17:

These pages are for the children to do on their own. You need not use all these pages at this time. They can be used later for review of the idea of equivalent sets.

Before giving the children these pages to do on their own, be sure that they have had experiences of matching elements of sets by connecting the elements of one set with the elements of a second set by drawing a line segment connecting the two elements. Using pieces of yarn on a flannel board to connect elements of two sets may be a quite different experience for some children from connecting the elements of two sets by a drawn line segment. Chalkboard work, where elements of two sets are connected with a line segment will help prepare pupils for these exercises.

Have the children open their books to Page 10. Ask them to cover the bottom part (bottom half) of the book with one hand or with a piece of paper. Read the title. Call attention to the two sets at the top of the page and the lines connecting the trees and the monkeys. Ask,

Are the two sets equivalent? (Yes.)

Direct the children to draw a ring around "yes".

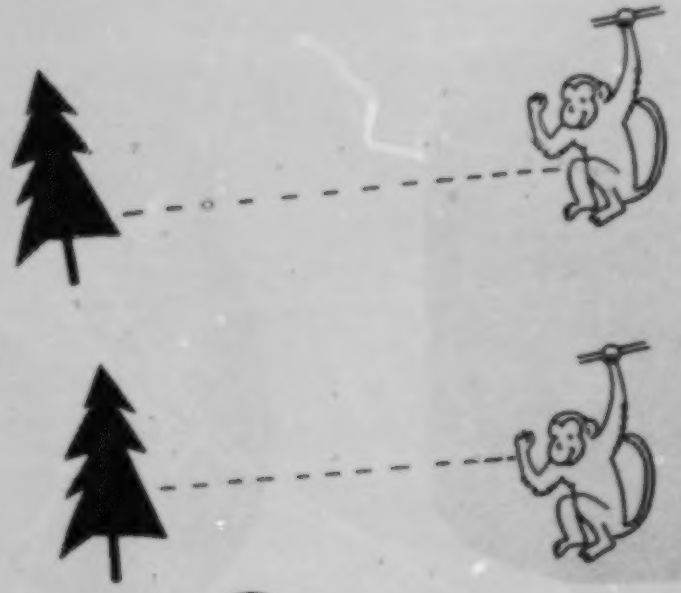
Now direct the children's attention to the two sets at the bottom of the page, covering the top picture. Guide the children as they match the members of the set of umbrellas with the members of set of girl (s) by drawing lines connecting the elements to the two sets. Ask,

Are the sets equivalent? (No.)
Why? (Because the elements cannot be matched.)

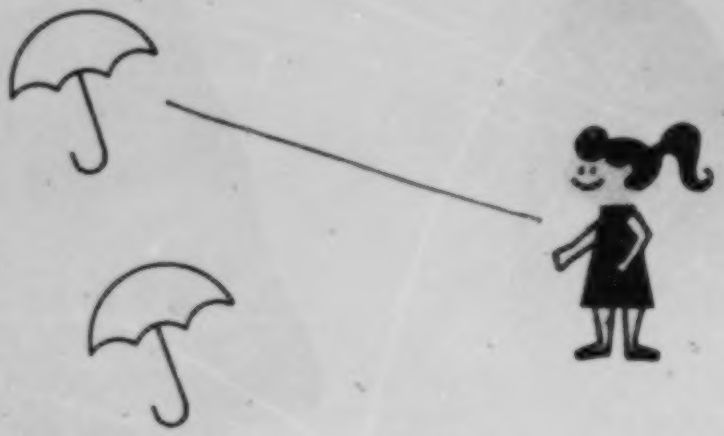
Guide the children to circle "no".

Continue with the same procedure for Pages 11-17. When you come to Page 12, call their attention to the dashed line that is already drawn. Have the children go over this dashed line together before starting the rest of the activity.

Are the sets equivalent?

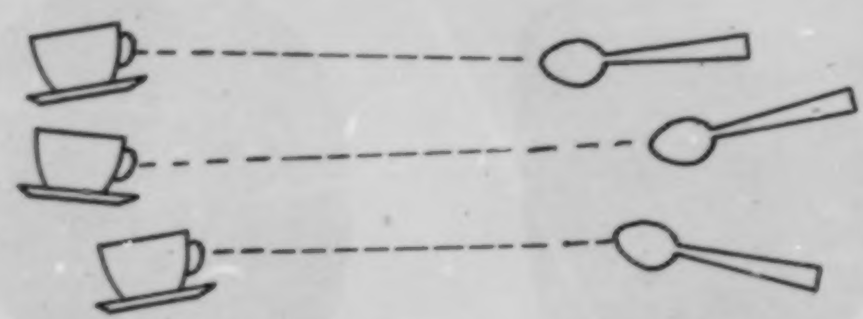


[✓ Yes] No

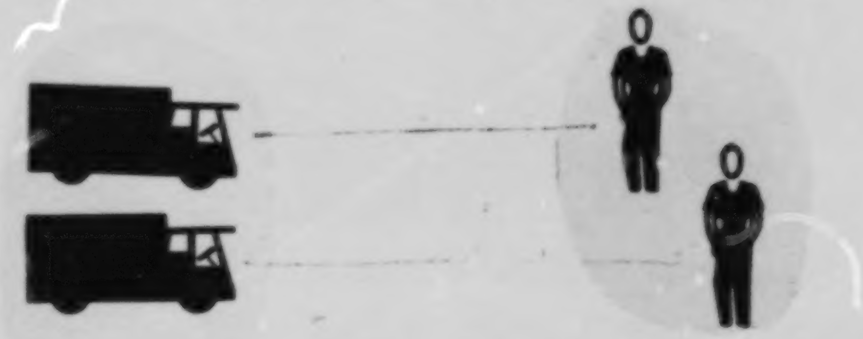


Yes [✓ No]

Are the sets equivalent?



[✓ Yes] No



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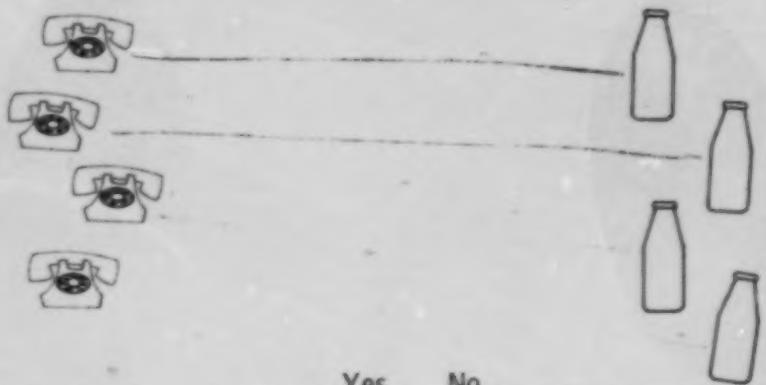
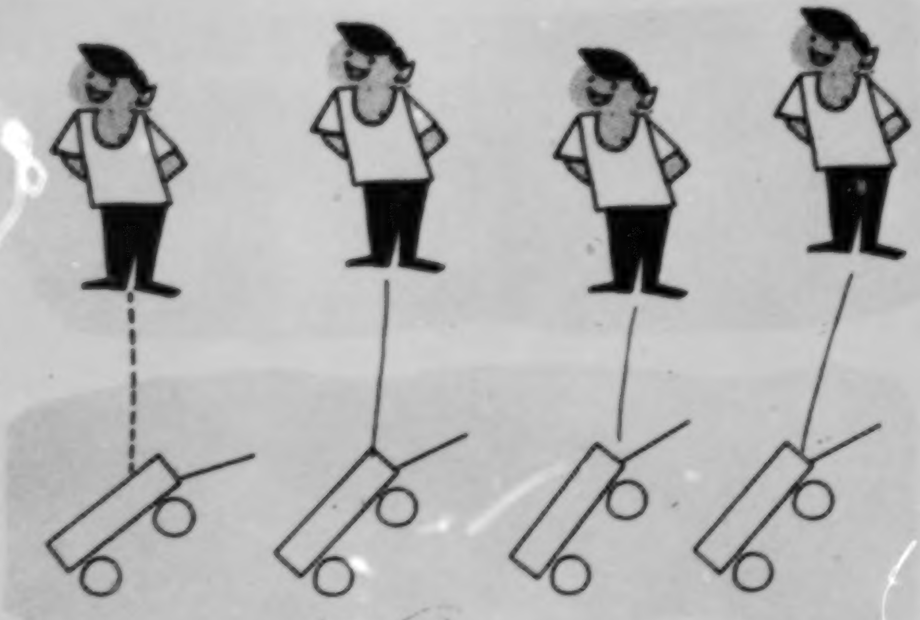
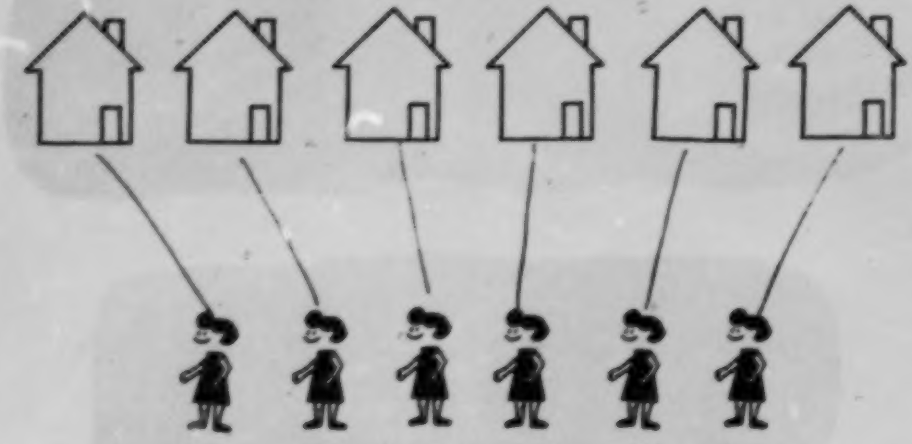


Yes No

Are the sets equivalent?

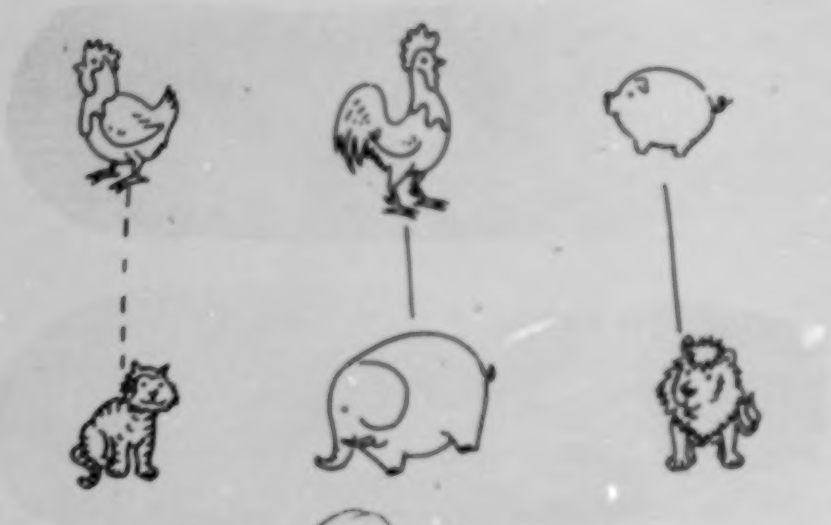


[✓ Yes] No

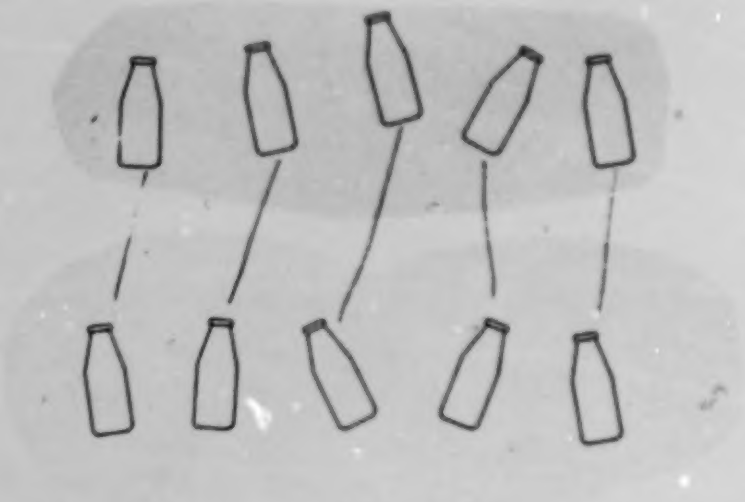


[✓ Yes] No

Are the sets equivalent?

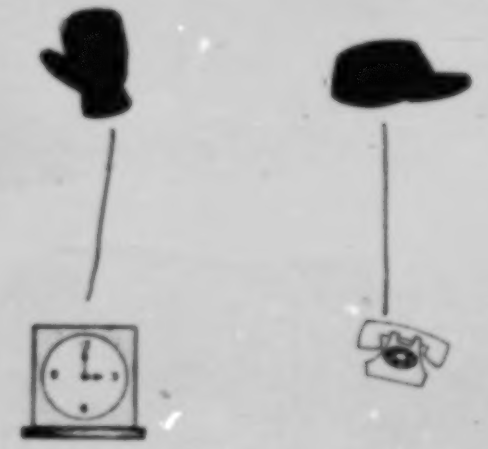


[✓ Yes] No

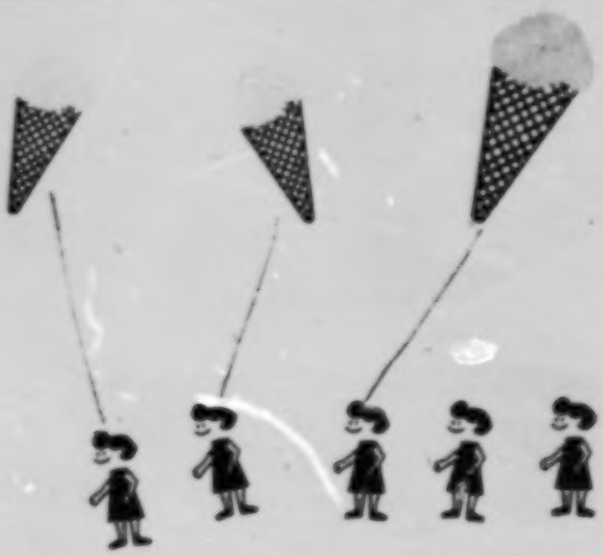


[✓ Yes] No

Are the sets equivalent?

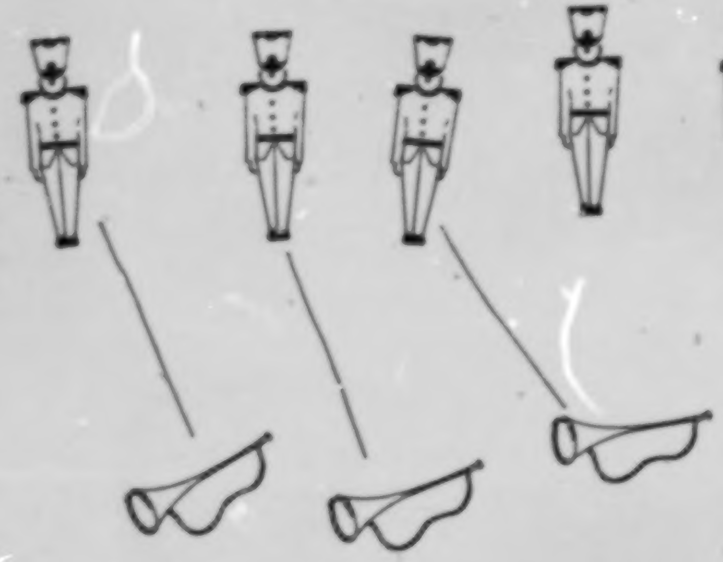


[✓ Yes] No

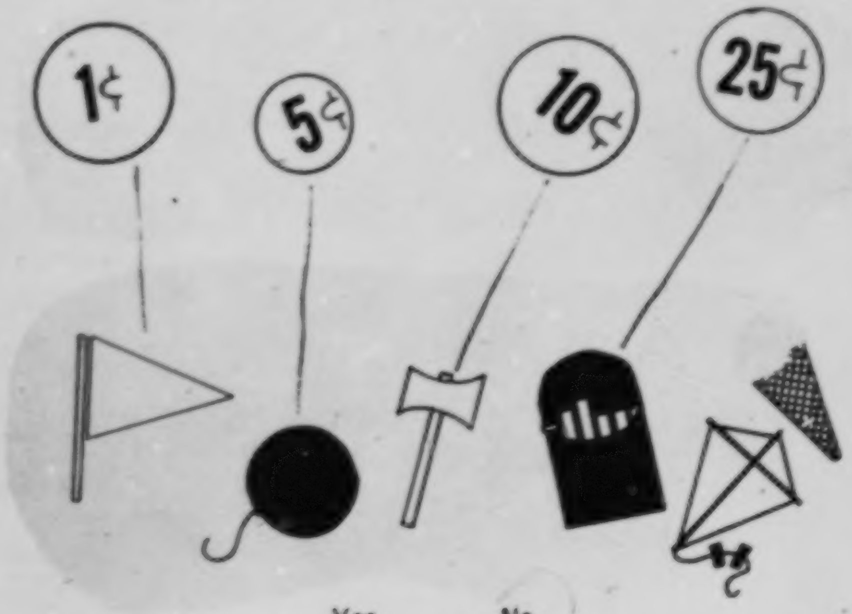


Yes No

Are the sets equivalent?



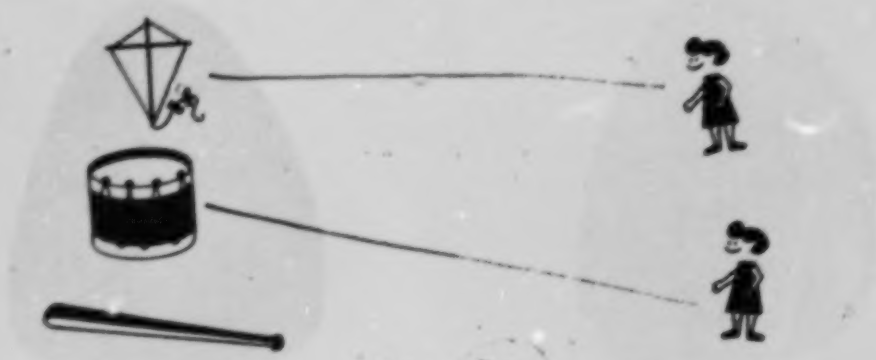
Yes [✓ No]



Yes No

Are the sets equivalent?

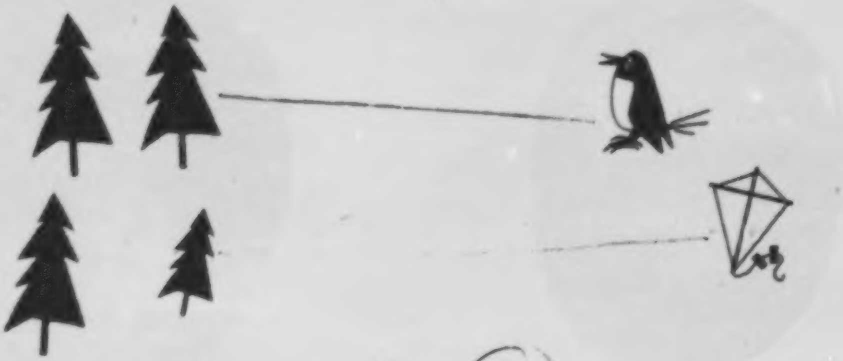
Yes No



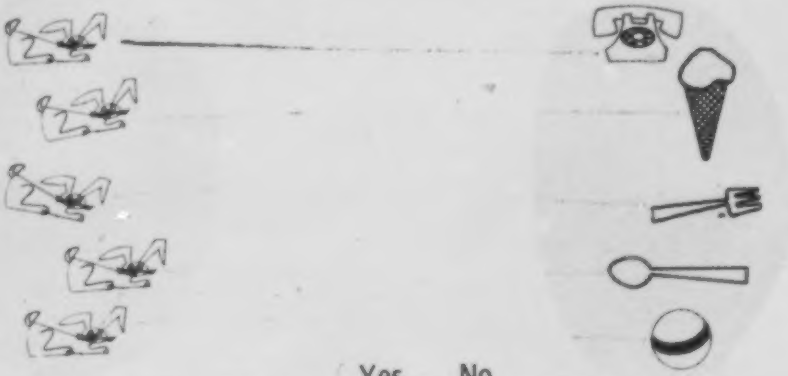
Yes [✓ No]

Yes No

Are the sets equivalent?



Yes [✓ No]



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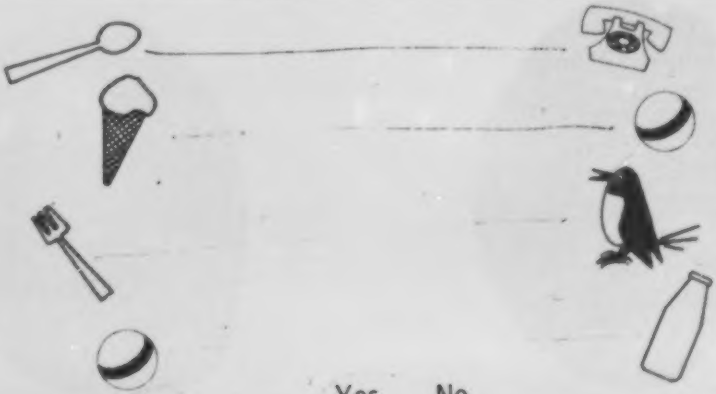


Yes No

USING THE PUPIL'S BOOK, pages 18-21:

Page 18 is a teaching page. Display your book in front of the pupils and make sure that each child his book open to the correct page. Read the title, pointing to the words as you read. Tell the children that they are to draw a set of rings in the space provided (point to the space) which matches the set of blocks. Say,

We draw a line from one of the blocks to the space below.
Then draw a ring (O) at the end of the line in the shaded space.

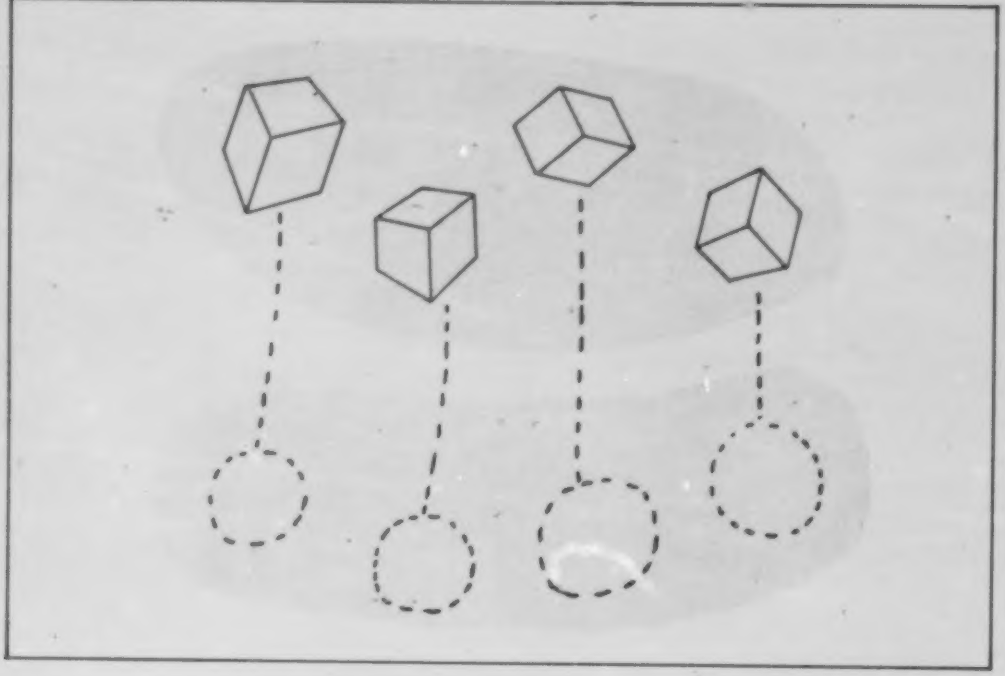
Have pupils mark over the dotted lines. After the same procedure has been followed for each block, then say,

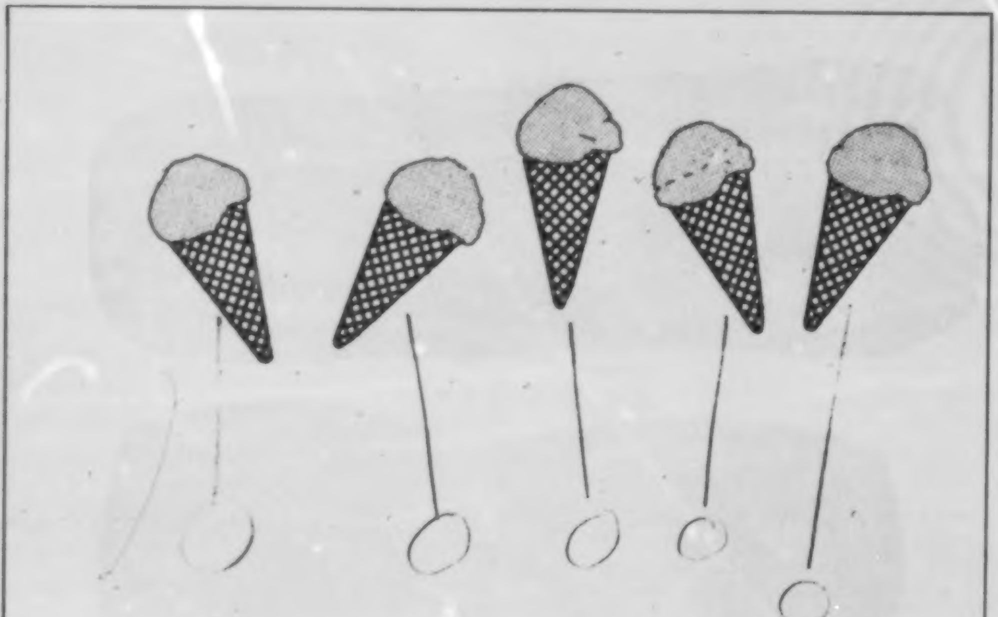
Now we have a set of rings which matches the set of blocks.

Do the same for the set of ice cream cones; then they can work Pages 19-21 on their own.

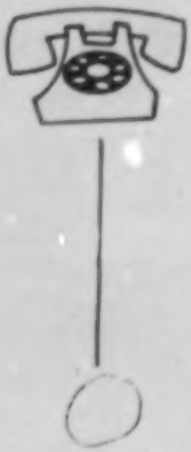
Show equivalent sets

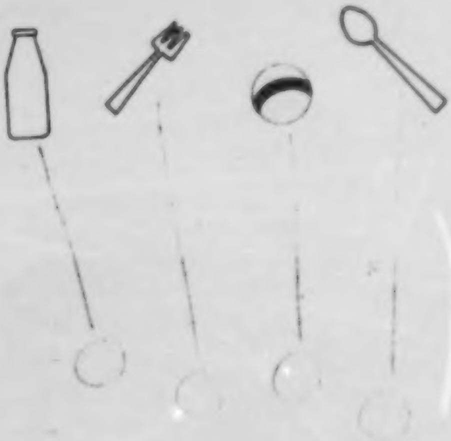
Teaching Page



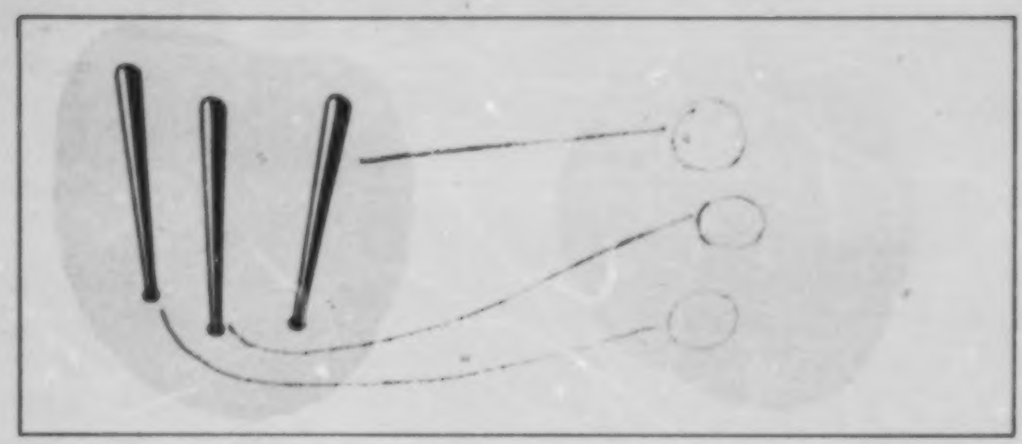


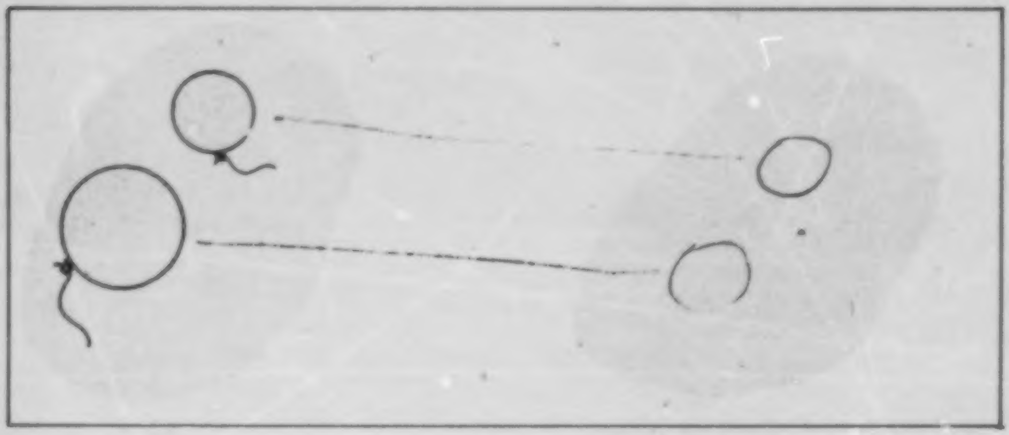
Show equivalent sets

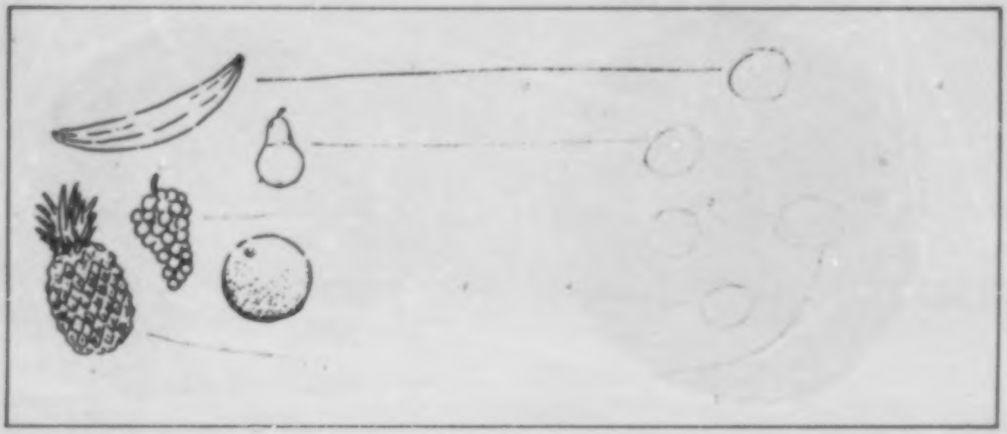




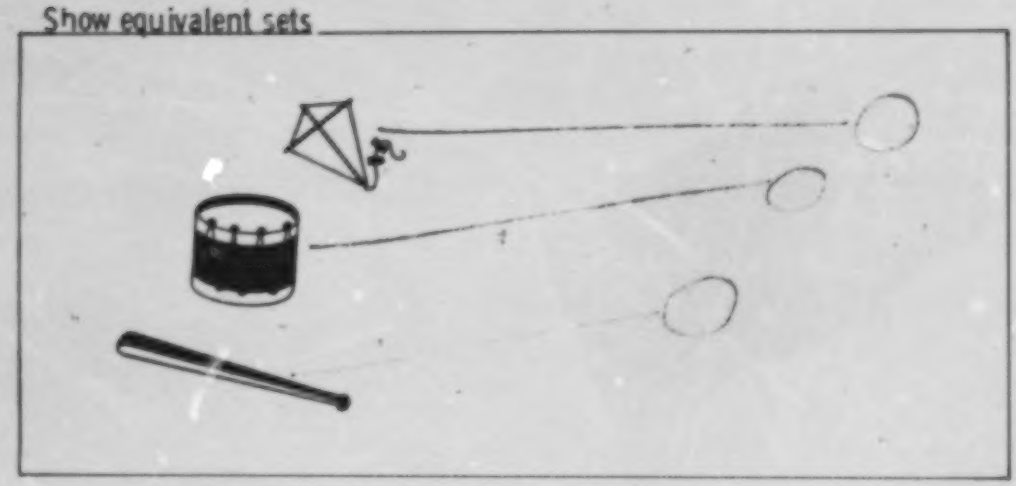
Show equivalent sets

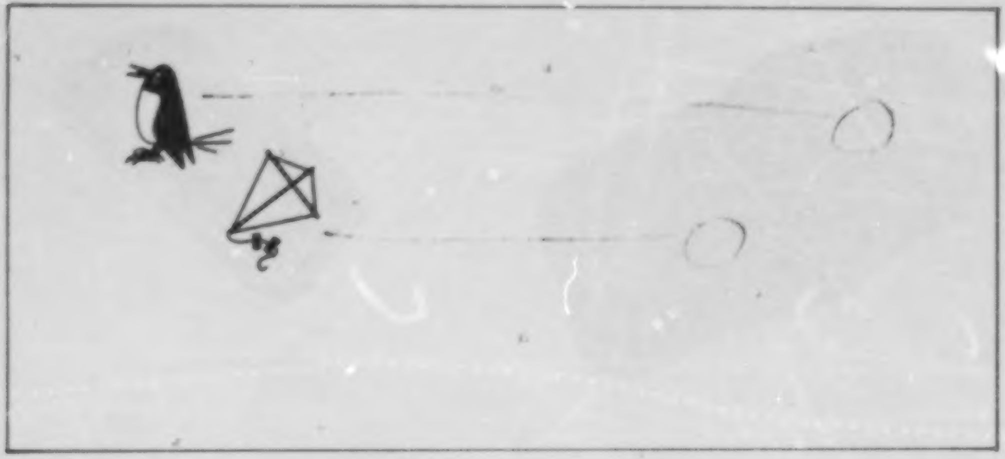


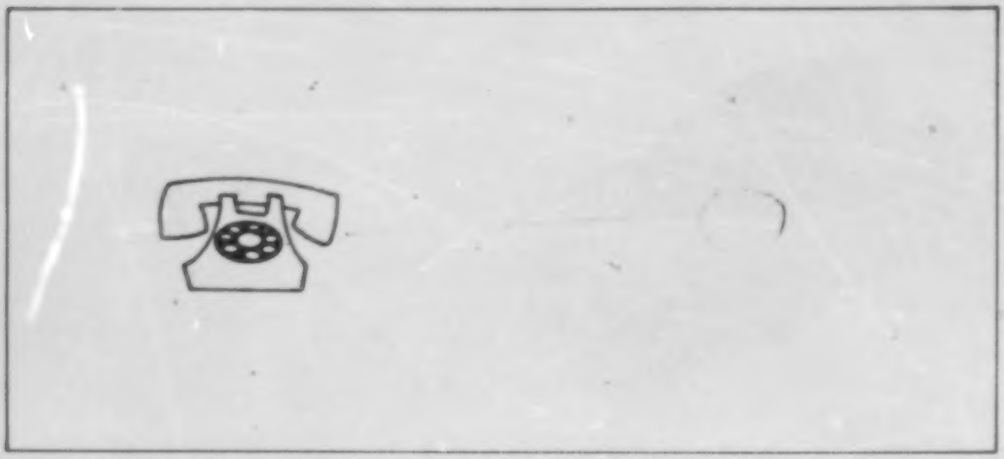




Show equivalent sets







1-4. COMPARISON OF SETS (MORE THAN)

OBJECTIVE: To introduce the idea that a set may have more or fewer members than another set.

VOCABULARY: More than.

BACKGROUND NOTE:

This lesson and the next one continues the work on pairing members of sets. If sets are not equivalent, we find that one set has more members than another, and that the latter has fewer members than the former. This is preparation for the concept of inequality of numbers. We shall later say that a number is greater than another if a set corresponding to the former has more members than a set corresponding to the latter. We shall say that a number is less than another if a set corresponding to the former has fewer members than a set corresponding to the latter. We use the terms "greater than" and "less than" for numbers, and the terms "more than" and "fewer than" for sets. These distinctions need not be emphasized with the pupils, but you should be reasonably consistent in your use of these words in this way.

MATERIALS: Flannel cut-outs, flannel board, buttons, clothes pins or other small objects, pictures of four story-book characters (Goldilocks and the Three Bears).

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

You will want to introduce the concept of "more than" first, waiting days before considering "fewer than." Children find the latter much more difficult to grasp. Mastery of the concept of "fewer than" should not be expected of all children at this time. Other activities which intervene between this introduction and the extension to the more sophisticated idea of a number being greater than, equal to, or less than another as well as the work with union of sets will add considerably to the development of these concepts.

Place 2 sets of 3 cut-outs on the flannel board.

Are these sets equivalent? (Yes.)
How do you know this is so? (There are as many members in the first set as in the second.)

How can we show that these sets are equivalent? (Pair.)
Mary, use these pieces of yarn (or flannel) to show the pairings.

When the pairings are completed, remove the cut-outs and place two other sets with 3 and 2 members on the flannel board.

Do these sets match? (No.)
Bill, use the pieces of yarn to show the pairings. (Pause.)
Are these sets equivalent? (No, there is a member left over.) The first set has more.)
When we paired the members of these sets, we found we had a member left over.
We say that this set has MORE members than the other set.

Give each child a handful of buttons and clothes pins, or have these already placed in the individual boxes or envelopes. Include a greater number of buttons. Have the children describe the sets and ask that the members of the sets be paired on their desks. Offer no help except when the pairing idea is misunderstood.

Were you able to pair all the members of your set of buttons with all the members of your set of clothes pins? (No, there were some buttons left over.)
We say that your set of buttons has MORE members than your set of clothes pins.

Instruct the children to move the clothes pins to one side of the desk or place them inside the box or envelope provided. Hold up one finger.

Use your buttons to make a set that has more members than the set of fingers I am showing you.

After checking to make certain that each child has a set of two or more buttons in the center of his desk, compliment the children and repeat the same procedure using 2, 3, and 4 fingers. Whenever a child makes a mistake, the teacher should pair her fingers with the members of the set of buttons on the desk.

Are there more buttons than fingers? (No.)
What will you have to do so that your set of buttons has more members than my set of fingers? (Place more buttons in my set.)

USING THE PUPIL'S BOOK, pages 22-23:

Use these pages as teaching pages following the activities suggested for developing the idea of more than using physical objects. Display Page 22 in front of the class making sure each child has his book open to the correct page. Read the title, pointing to the words as you read. Now, point to the top box and say,

We have two sets of blocks.
Are there more blocks in one set than in the other?
How can we decide? (By matching.)

Have the students match blocks by tracing over the dotted marks.

There are two members left over in the top set, so the top set of blocks has more members.
Let us put an X on the line beside the set with more members.

Pupils can trace over the dotted X. Continue with other exercises, discussing each as you did with the physical objects. If one set does not have more members than the other as in the last exercise on Page 24, then instruct the children that they do not put a mark on either line.

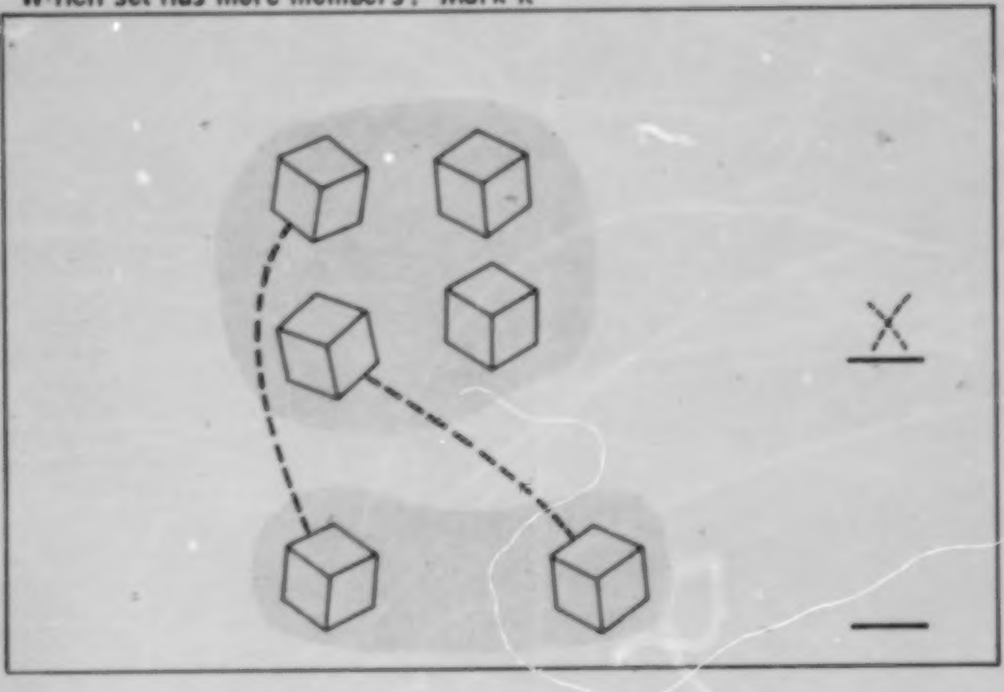
Caution the children that the sets of pictures on the second page are placed differently. Note how the pairing has been done to help decide which set has more members and that the question is answered by placing the X on the line below the set.

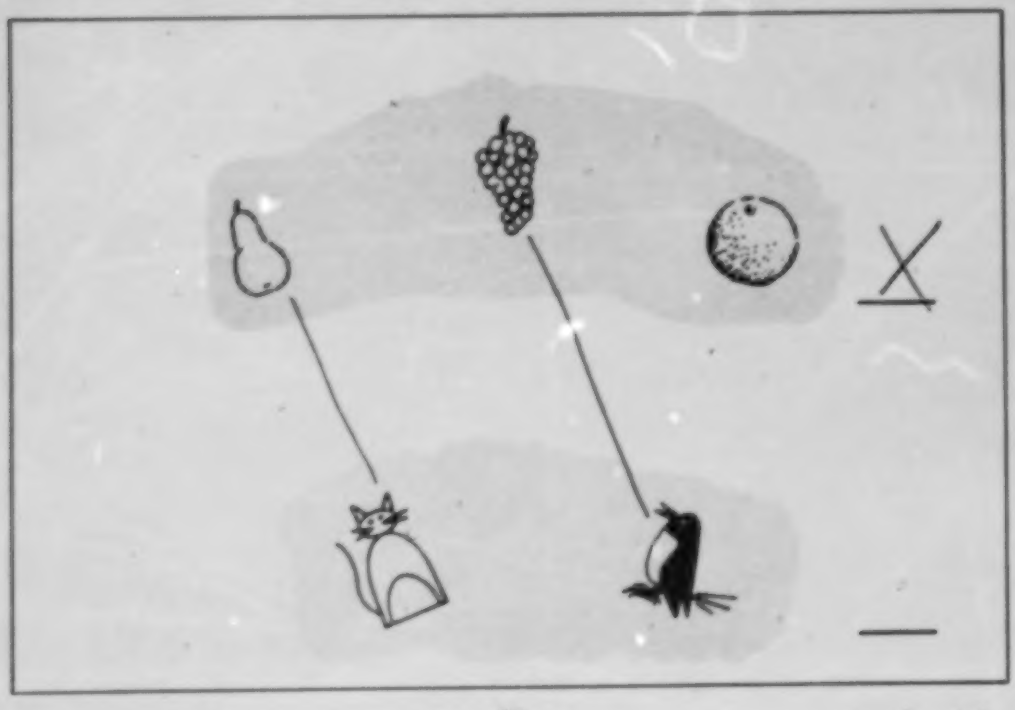
USING THE PUPIL'S BOOK, pages 24-29:

Children use these pages on their own. Read the title. Give instructions as above. Caution the children that if one set does not have more members than the other set, then they do not put a mark on either line.

Teaching Page

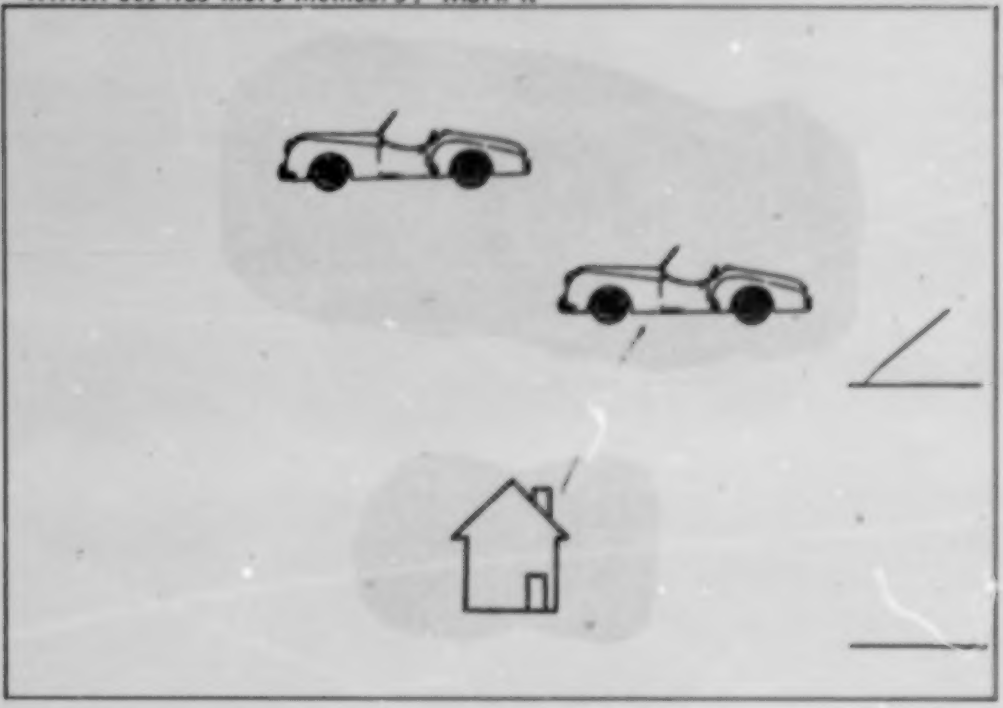
Which set has more members? Mark it

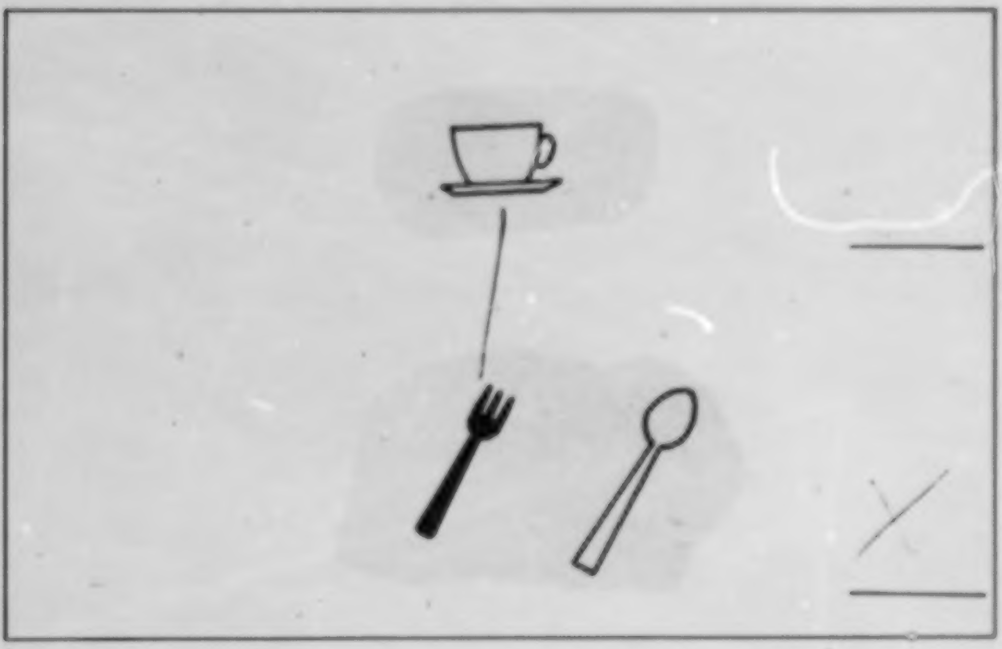




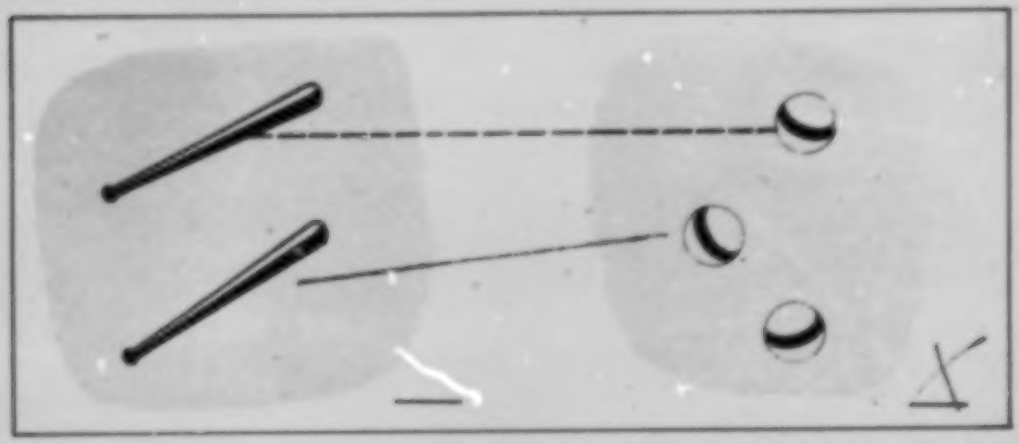
Teaching Page

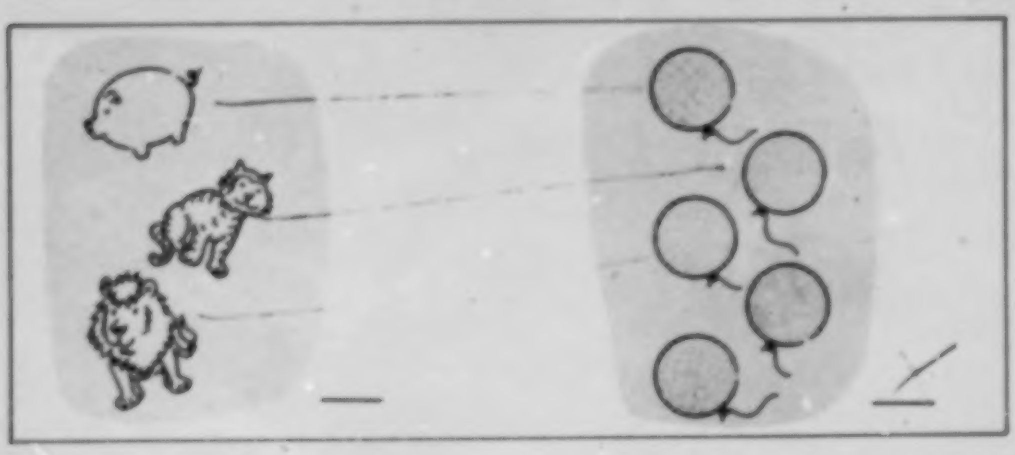
Which set has more members? Mark it

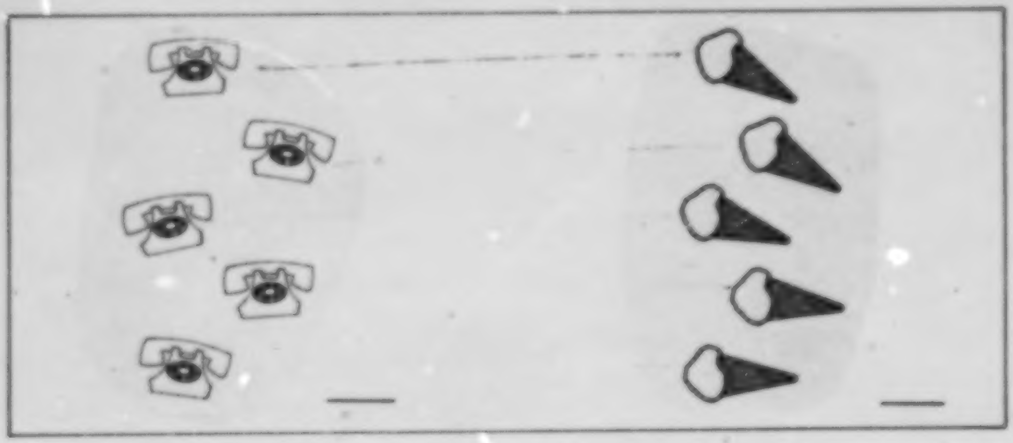




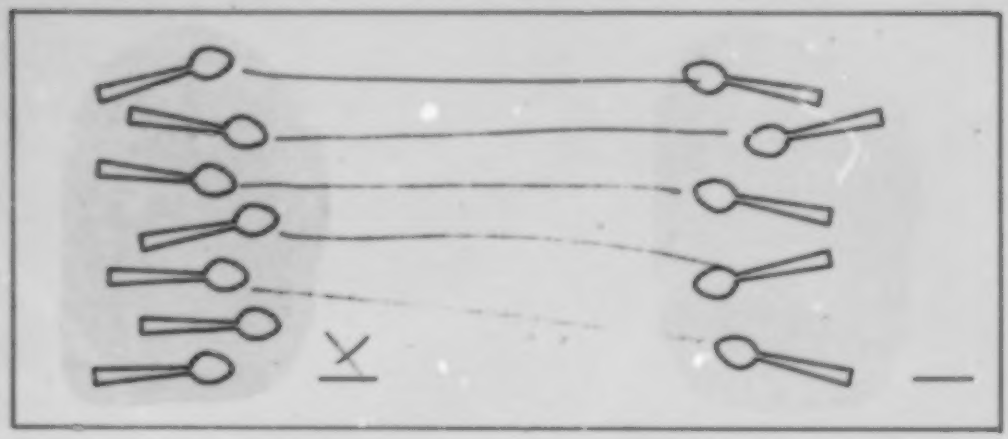
Which set has more members? Mark it

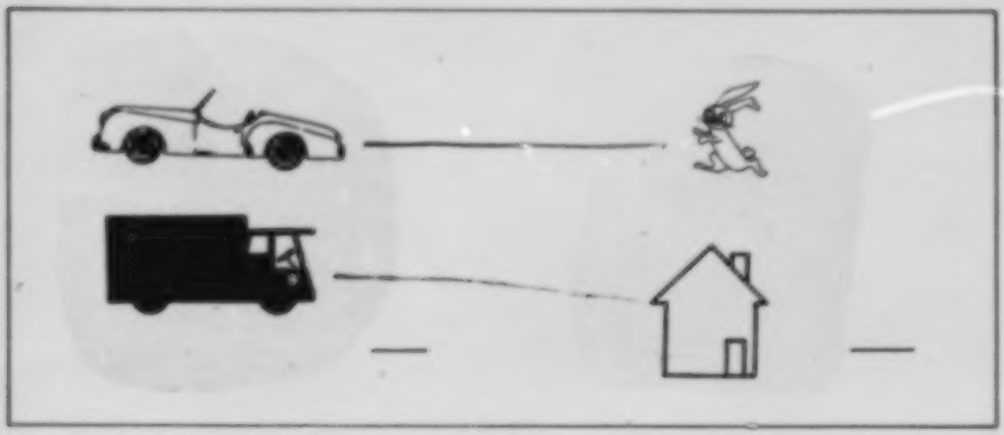


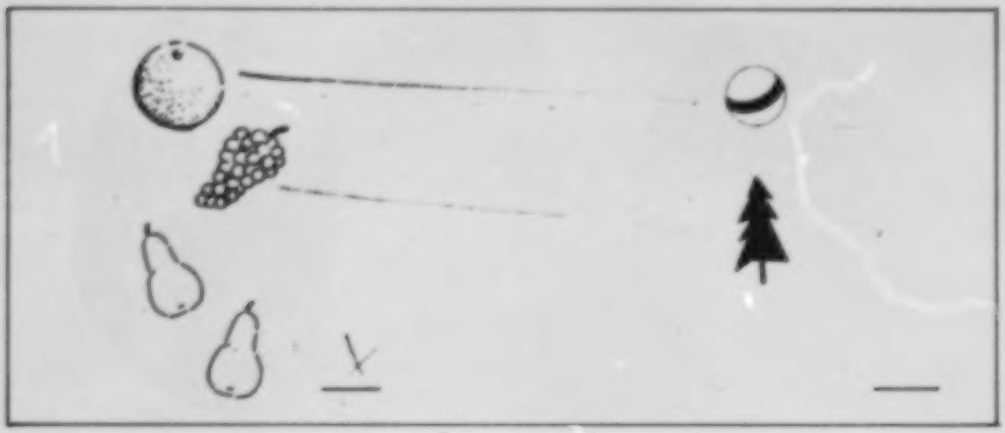




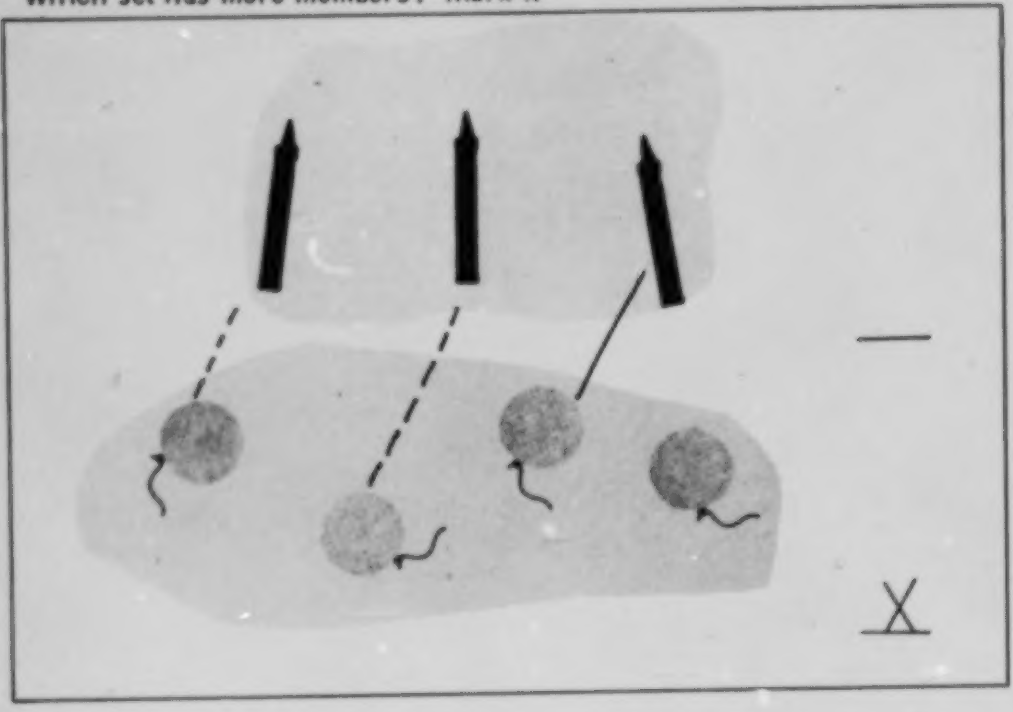
Which set has more members? Mark it

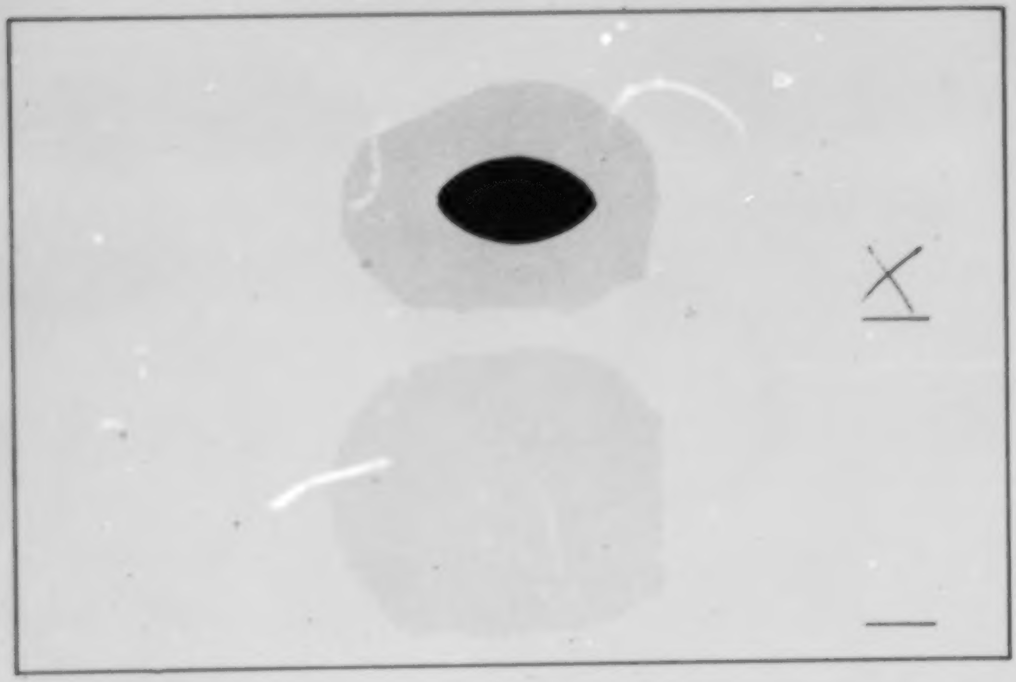




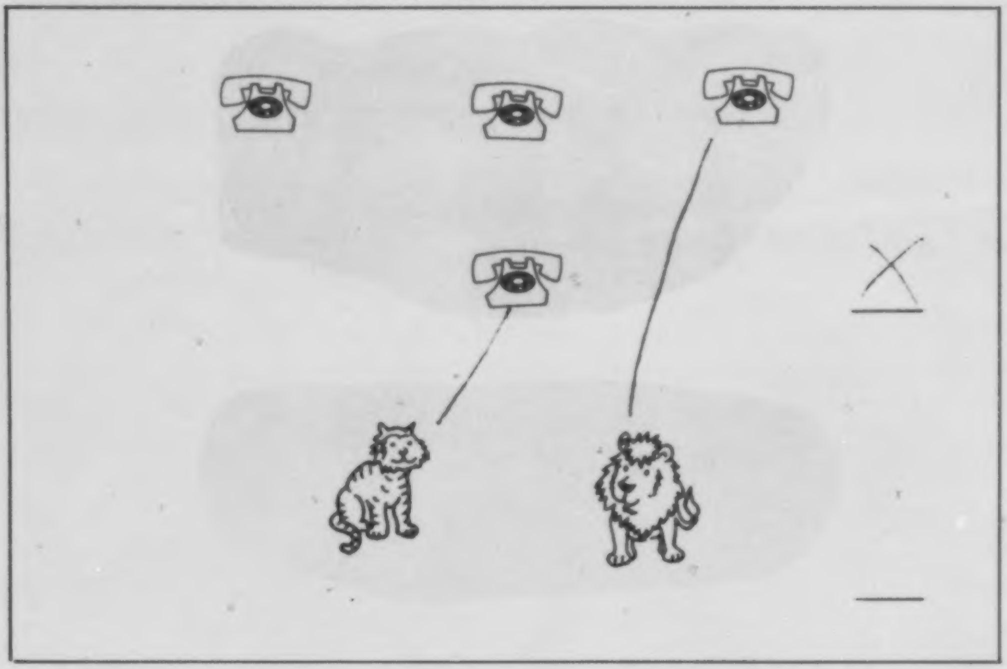


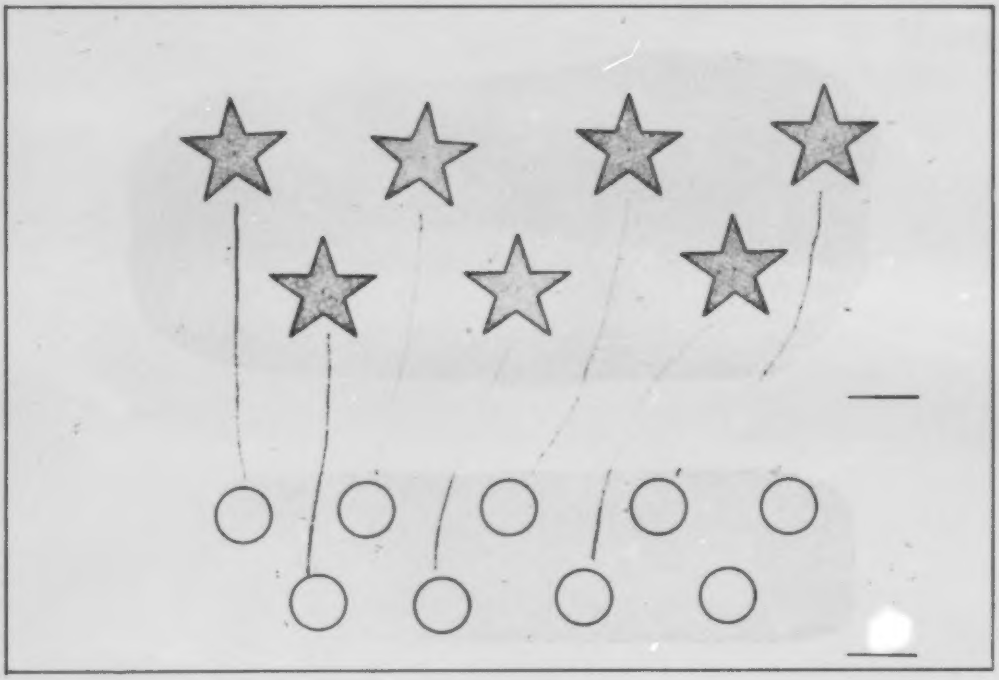
Which set has more members? Mark it



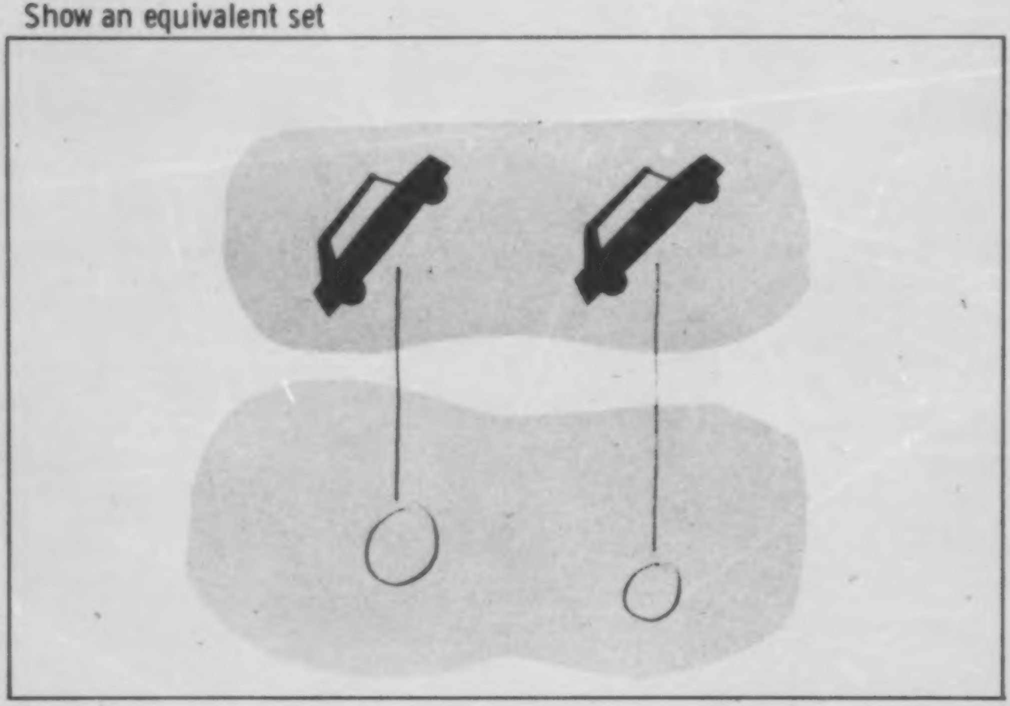


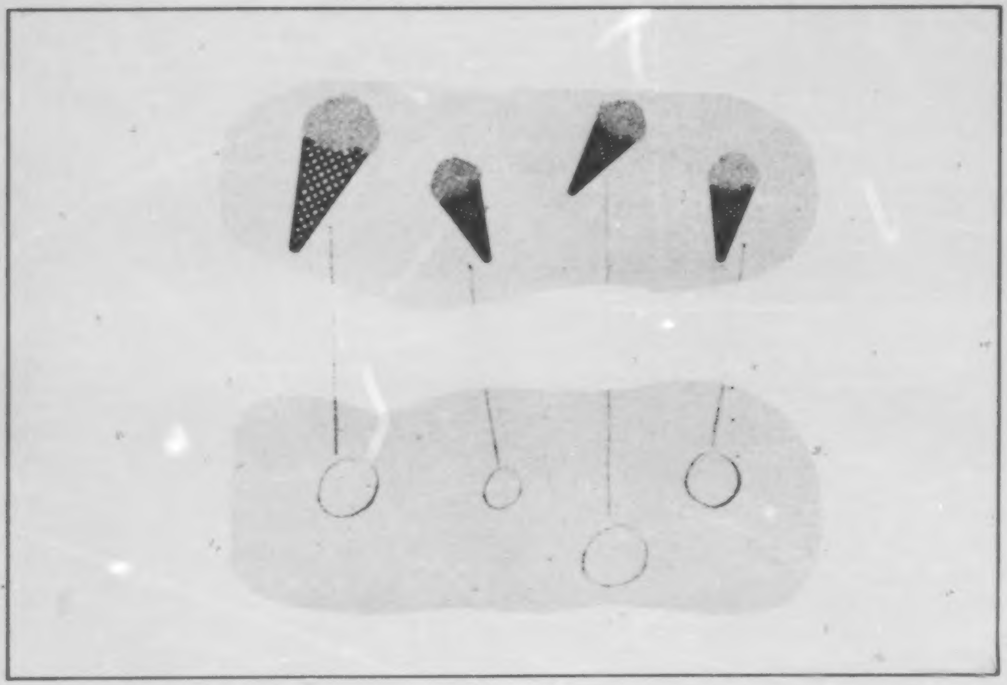
Which set has more members? Mark it



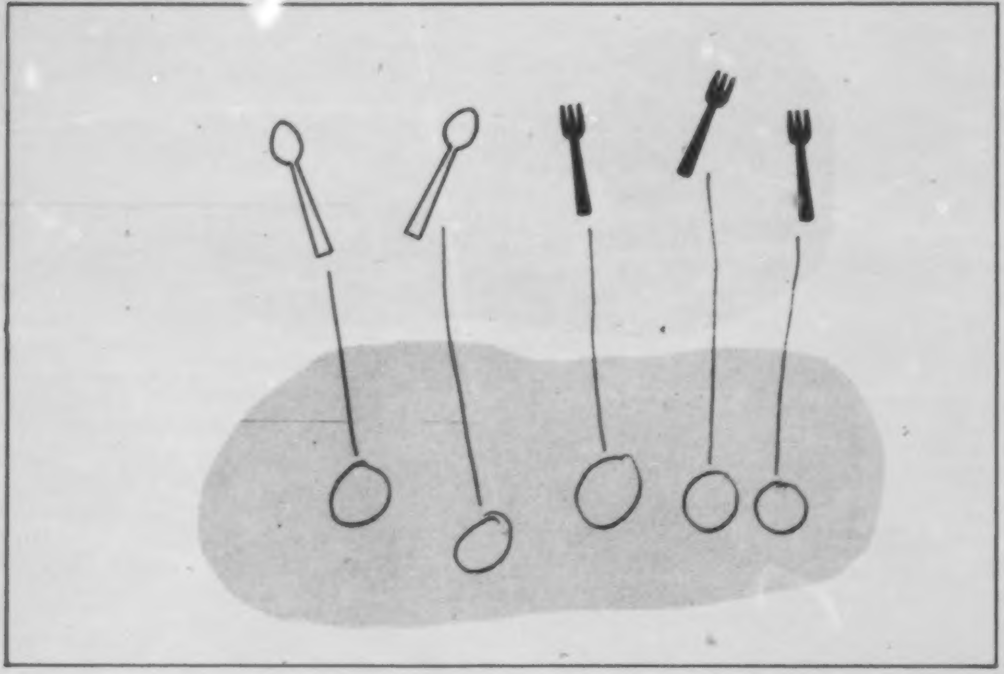


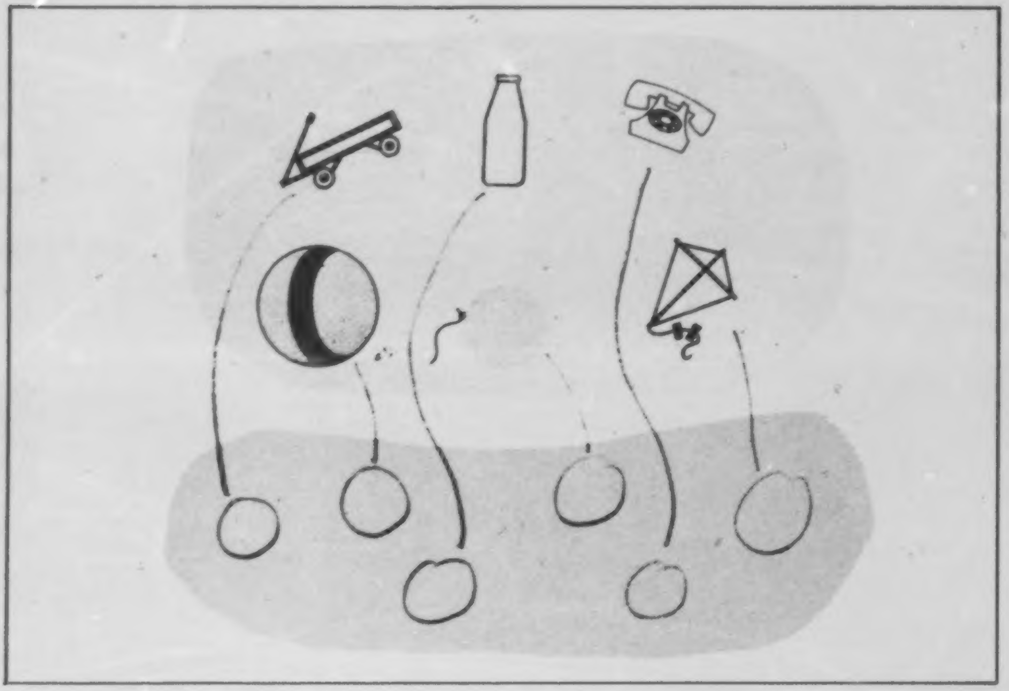
Show an equivalent set





Show an equivalent set





USING THE PUPIL'S BOOK, pages 30-32:

Page 30 is a teaching page. Display the page in front of the pupils and make sure each child has his book open to the correct page. Read the title. Explain that they are to draw a set which has more members than the given set. Suggest that they first draw a set of rings equivalent to the given set, then they can add as many more rings as they wish. Say,

> Let us draw a line from the bicycle to the space below. (Children should trace over the dotted marks.)
> Draw a ring at the end of the line.
> Now, we have a set of rings which is equivalent to the set of bicycles.
> We want the set of rings to have more members, so add more rings.

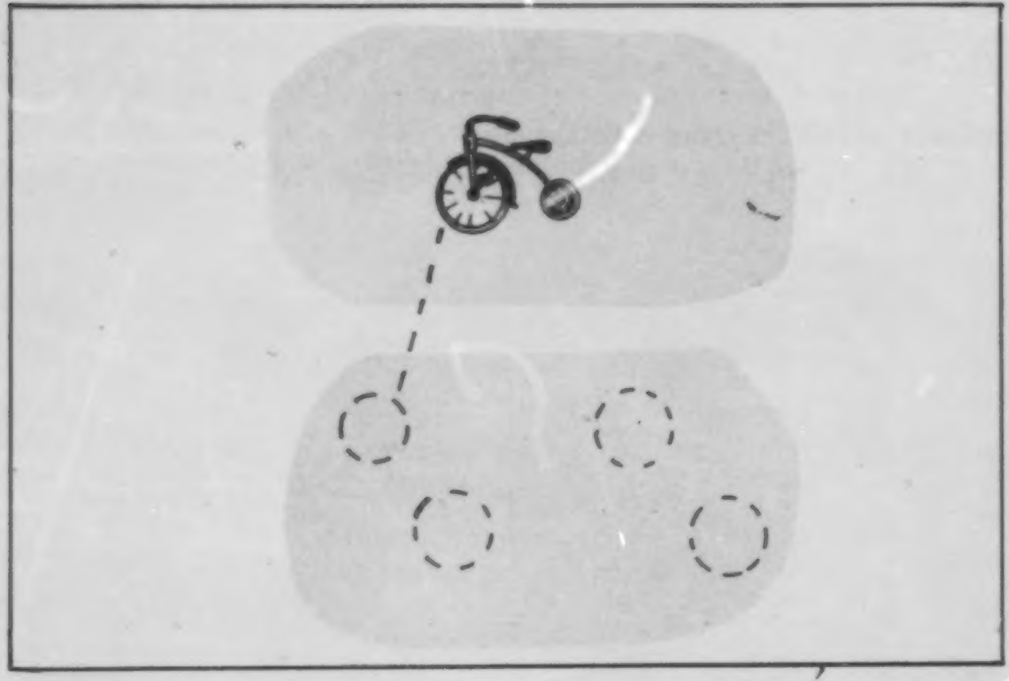
Repeat the same procedure with the set of coins.

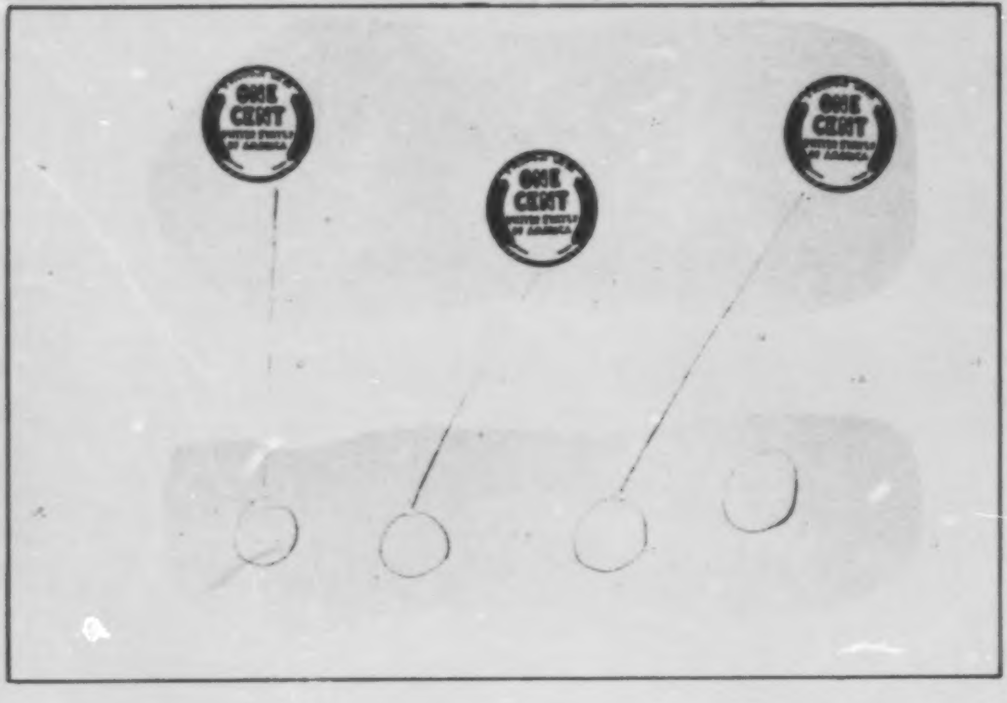
Pages 31 and 32 are for independent work.

Page 33:

This page is a review of the idea of equivalent sets. Read directons to the children.

Show a set with more members

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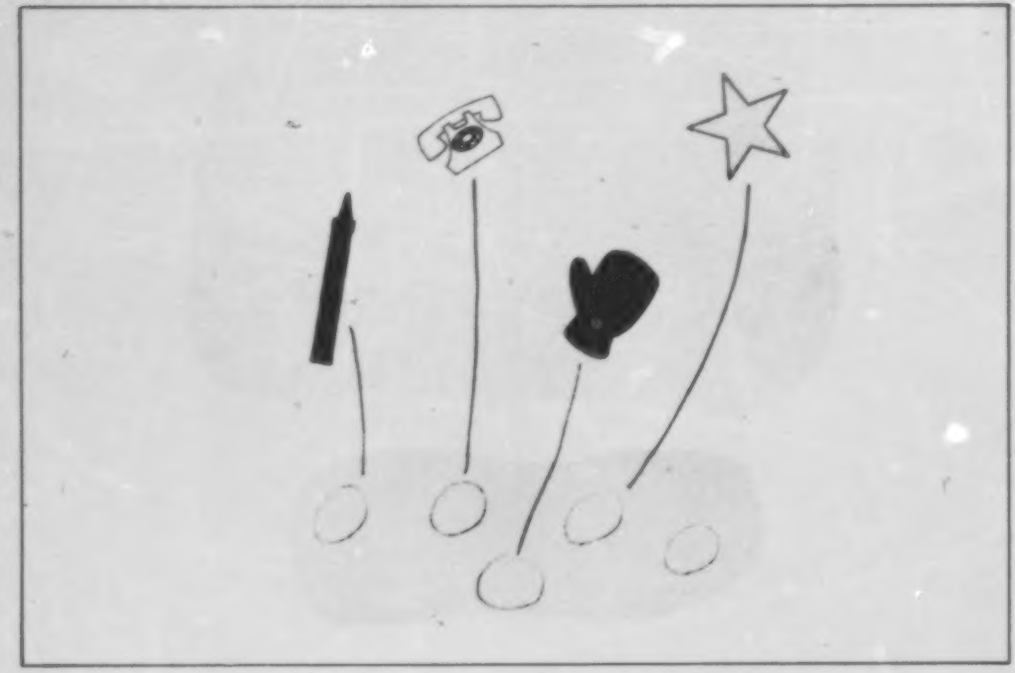
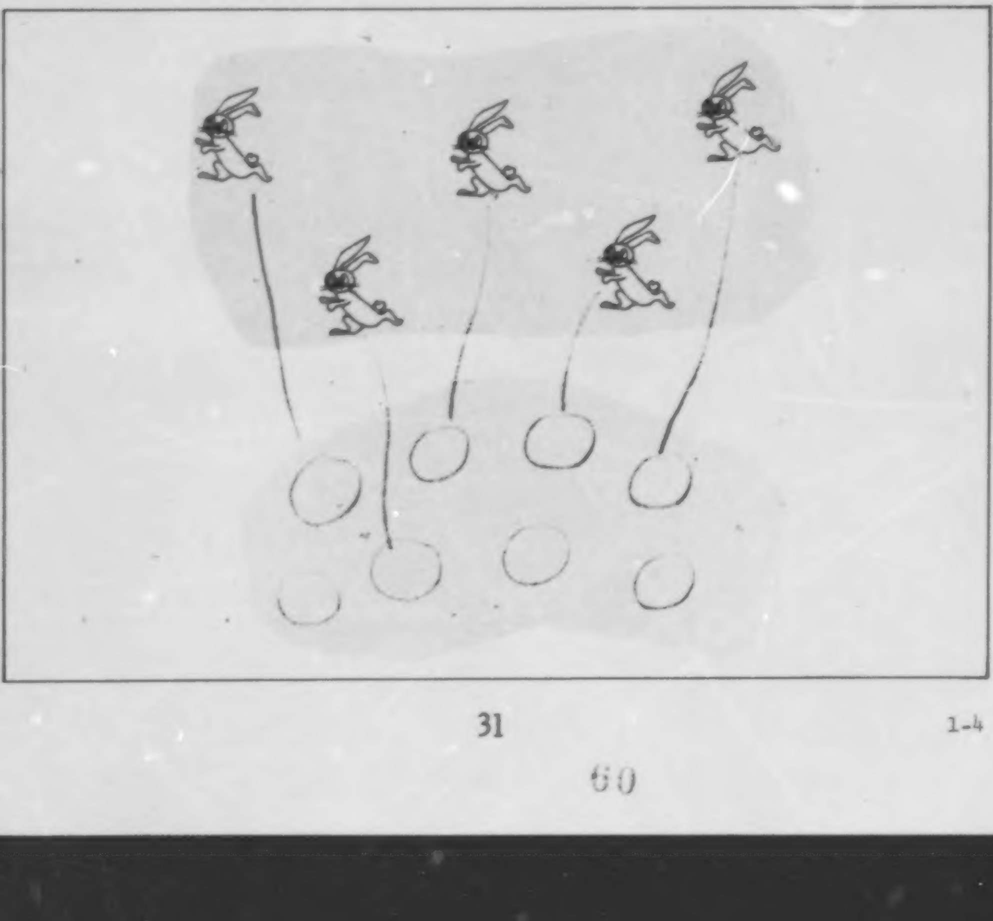
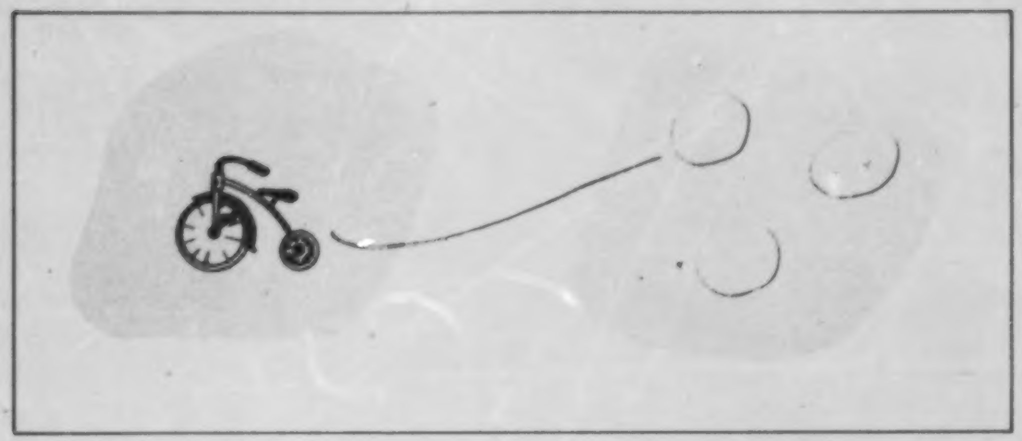
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA

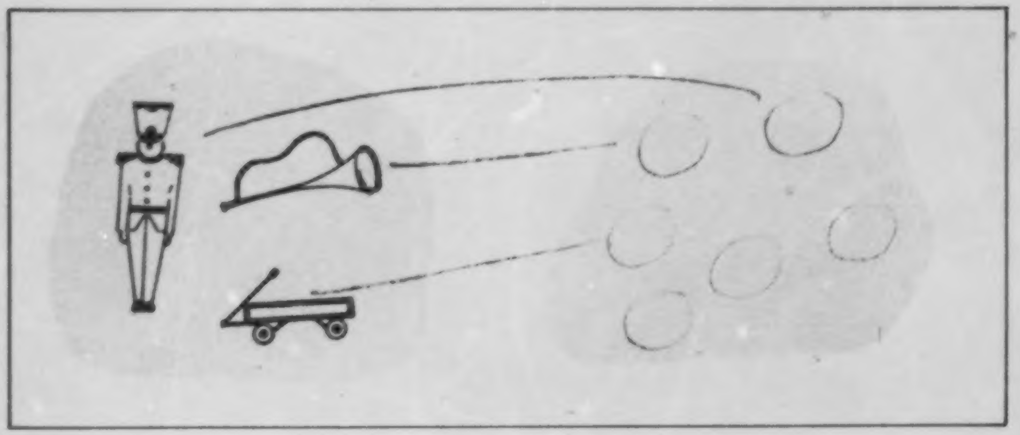
Show a set with more members

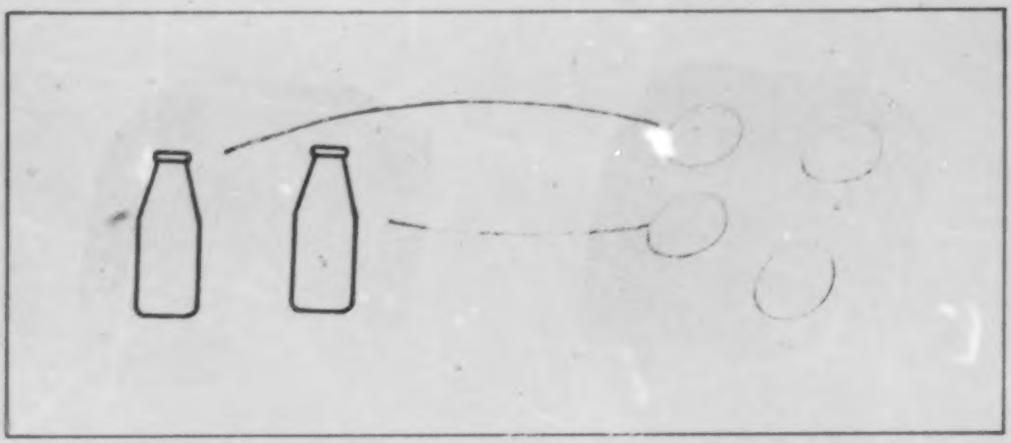




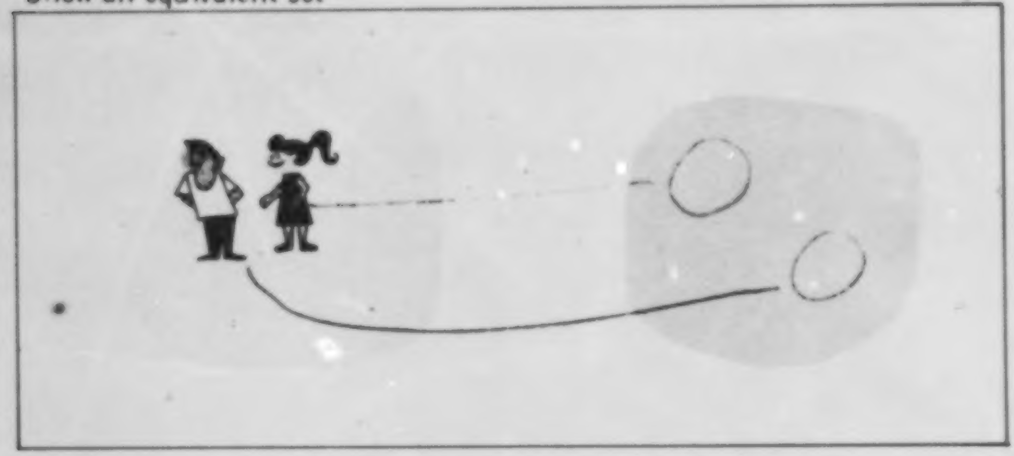
Show a set with more members

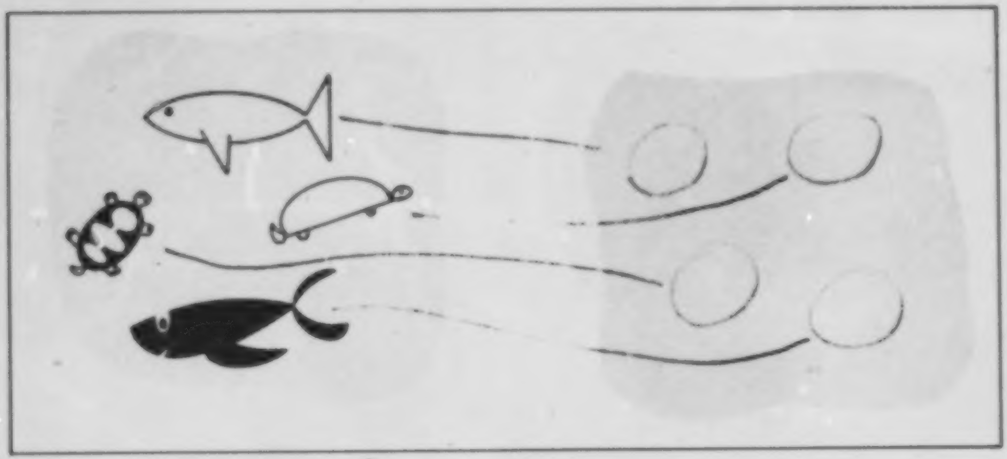


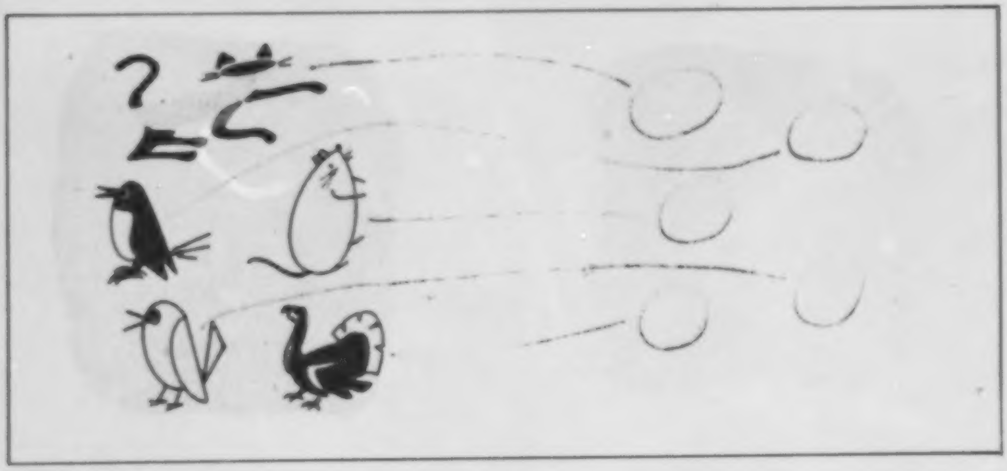




Show an equivalent set







1-5. COMPARISON OF SETS (FEWER THAN)

OBJECTIVE: To introduce the idea that a set may have fewer members than another set.

VOCABULARY: Fewer than.

BACKGROUND NOTE:

See 1-4, page 39.

MATERIALS: Flannel cut-outs, flannel board, buttons, clothes pins or other small objects.

SUGGESTED PROCEDURES:

PRE-BOOK ACTIVITIES:

Place 2 sets of 2 cut-outs on the flannel board.

Are these sets equivalent? (Yes.)
How do you know this is so? (There are as many members in the first set as in the second set.)
How can we show that these sets are equivalent? (Pair.)
John, use these pieces of yarn to show the pairing.

When the pairing is completed, remove the cut-outs and place two other sets with 1 and 2 members on the flannel board.

Do these sets match? (No.)
Ann, use the pieces of yarn to show the pairing. (Pause.)
Are these sets equivalent? (No, there is a member left over. The second set has more.)
Which set has fewer members. (The first one.)
Yes, the first set has fewer members.

Give each child a handful of buttons and clothes pins, or have these already placed in the individual boxes or envelopes. Include a greater number of buttons. Have the children describe the sets and ask that the members of the sets be paired on their desks. Offer no help except when the pairing idea is misunderstood.

Were you able to pair all the members of your set of buttons with all the members of your set of clothes pins? (No, there were some buttons left over. There aren't enough clothes pins.)

Which set has fewer members? (Set of clothes pins.)
Yes, the set of clothes pins has fewer members than the set of buttons.
There aren't as many clothes pins as buttons.

Instruct the children to move the clothes pins to one side of the desk or place them inside the box or envelope provided. Hold up two fingers.

Use your buttons to make a set that has fewer members than the set of fingers I am showing you.

After checking to make certain that everyone has a set of one or no buttons in the center of his desk, compliment the children and repeat the same procedure using 3, 4, and 1 fingers. Whenever a child makes a mistake, the teacher pairs her fingers with the members of the set of buttons on the desk.

Are there fewer buttons than fingers?
What will you have to do so that your set of buttons has fewer members than my set of fingers?

USING THE PUPIL'S BOOK, pages 34-37:

Pages 34-35:

Use these pages as teaching pages following the activities suggested for developing the idea of fewer than using physical objects.

Ask if there are fewer pennies than turkeys. Ask,

How can we decide?

After tracing the lines pairing members of the two sets put an X on the line beside the set with fewer members.

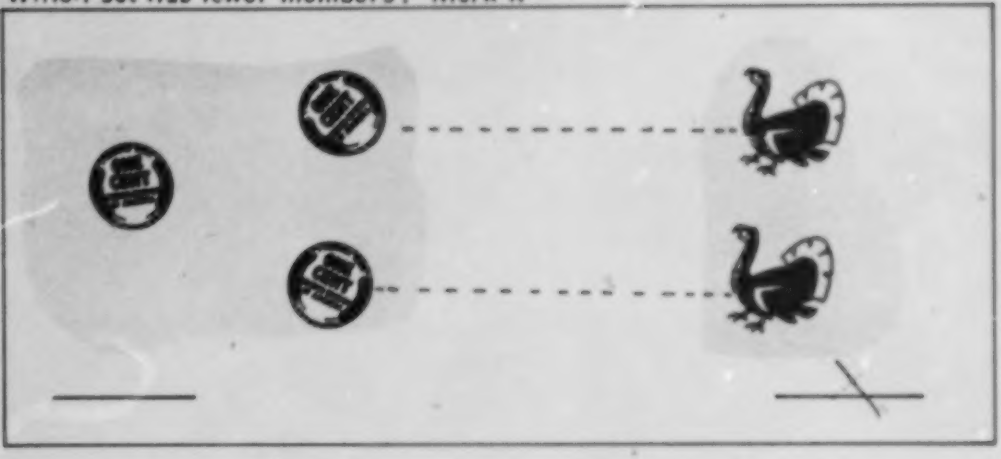
Continue with the other sets, discussing each pair of sets as you did with physical objects. If onse set does not have fewer members than the other as in the last example on Page 35, then instruct the children that they do not put a mark on either line.

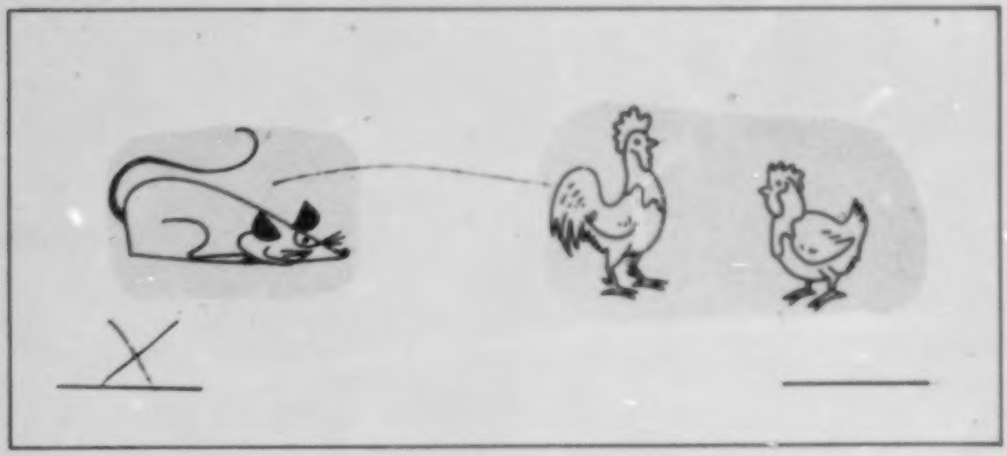
Pages 36-37:

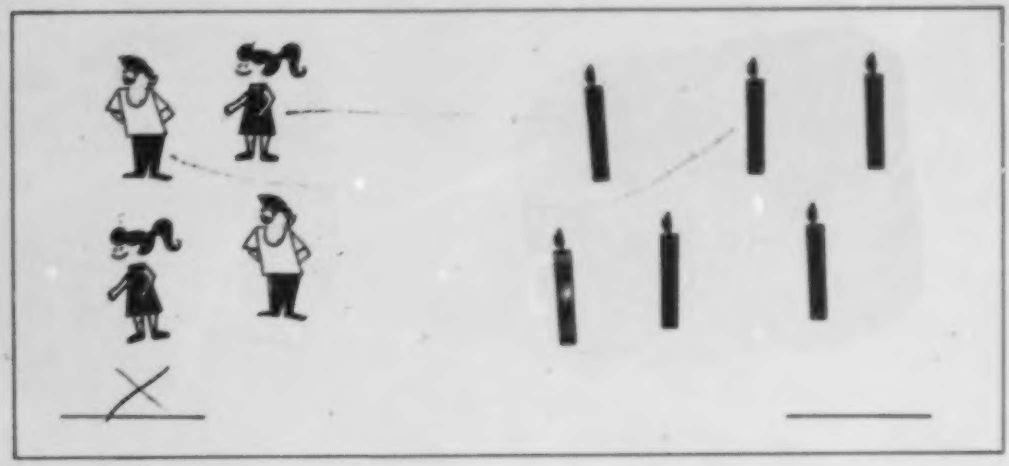
These pages are to be worked by the children on their own.

Teaching Page

Which set has fewer members? Mark it

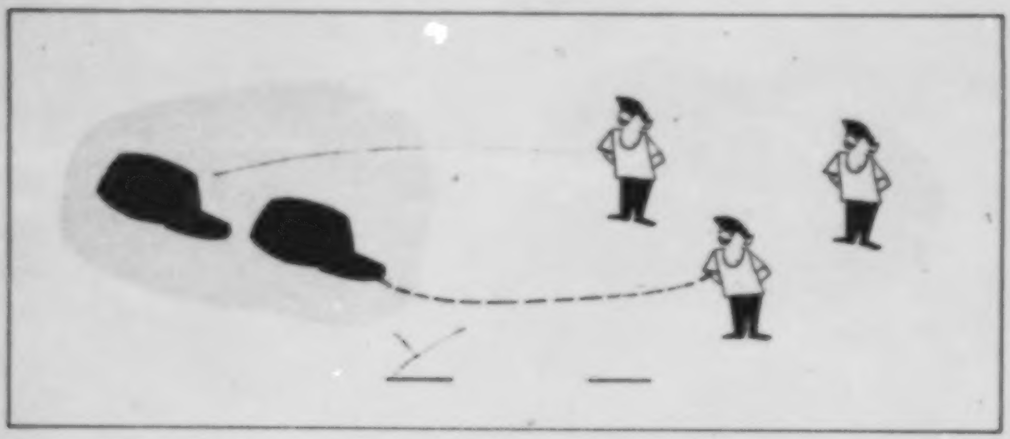


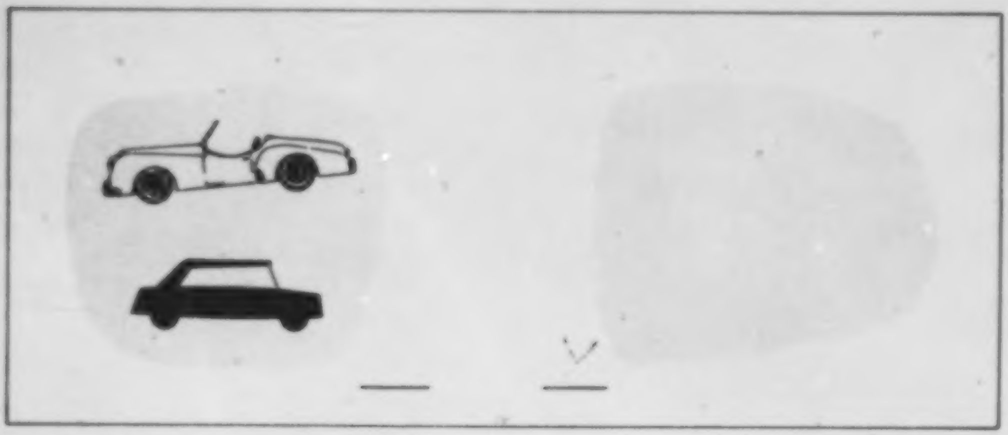


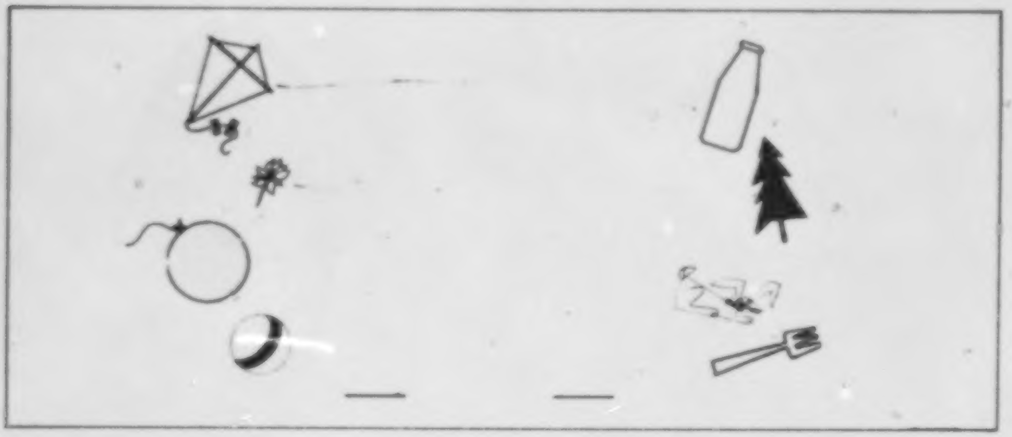


Teaching Page

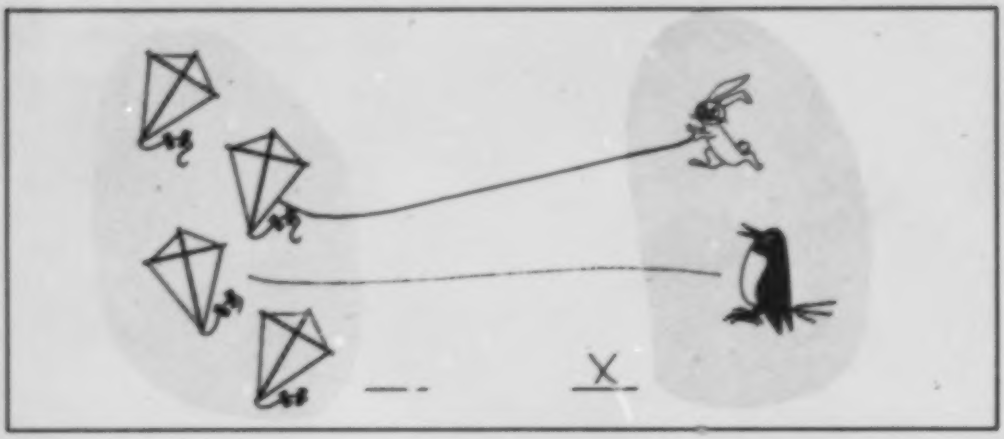
Which set has fewer members? Mark it

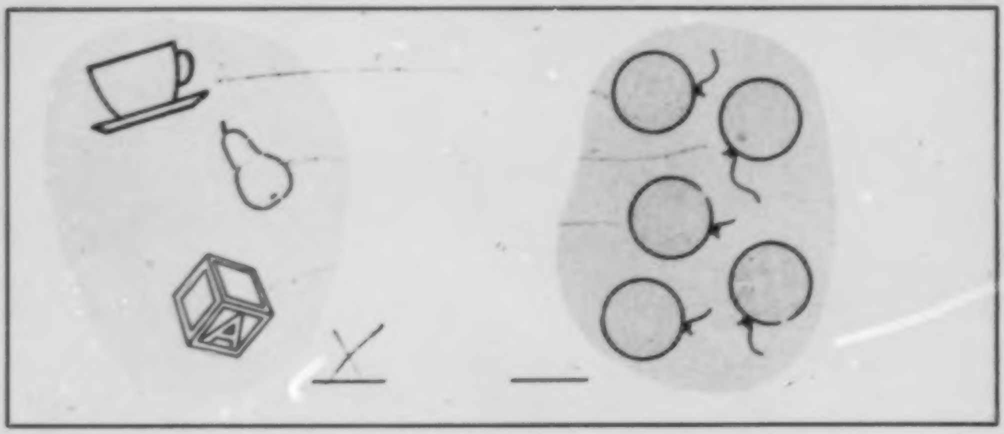


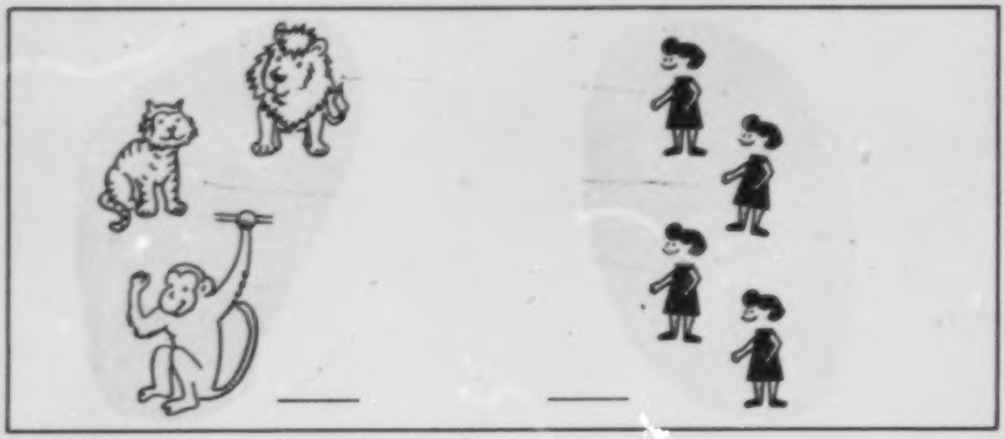




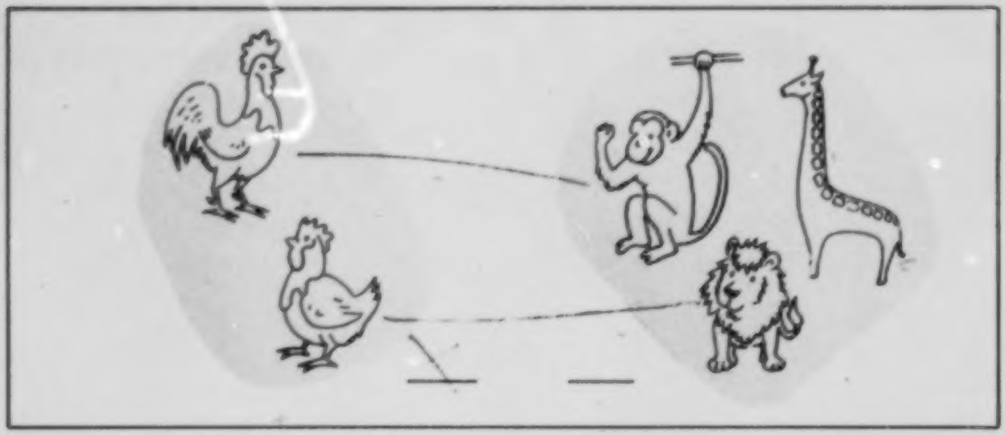
Which set has fewer members? Mark it

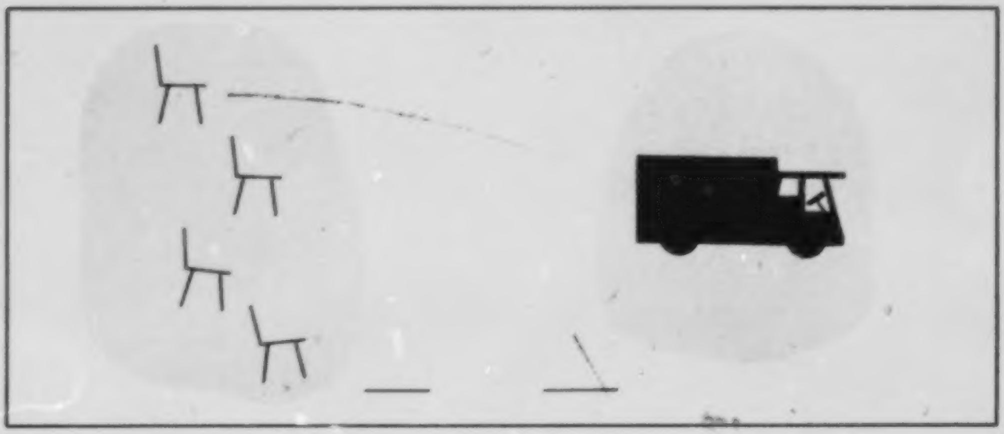


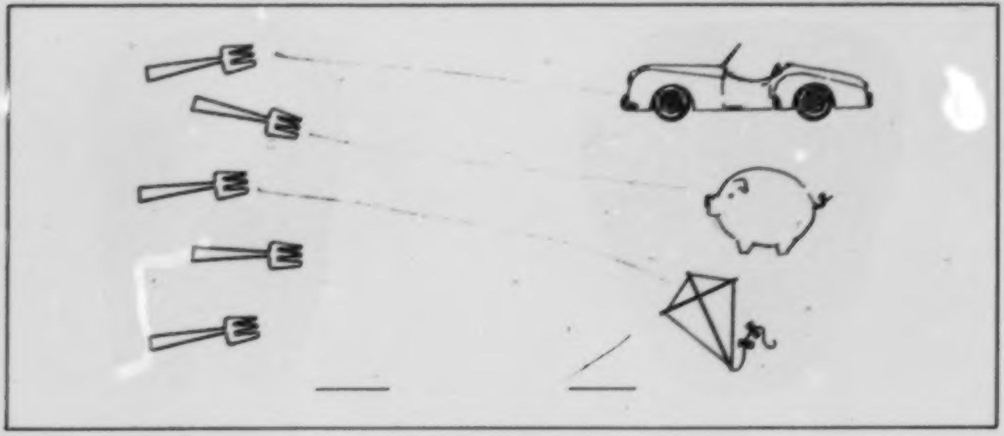




Which set has fewer members? Mark it







USING THE PUPIL'S BOOK, pages 38-39:

Page 38 is a teaching page. After the usual procedures of displaying your book, making sure each child has his book open to the correct page, and reading the title, say:

> *We want to draw a set of rings in this space* (*point to the space*) *so that the set of rings has fewer members than the set of balls.*

Then suggest that they mark at least one ball with a large X. Then they can draw a set of rings which is equivalent to the set of balls remaining.

FURTHER ACTIVITIES:

1. Ask three children to stand by the flannel board. Ask another child to give a story character picture to each of the three children. (There should be four pictures.)

> *Is there a child for each character in the story.* (No.)
> *Are there more children than there are story characters?* (No.)
> *Are there as many children as there are story characters?* (No.)
> *Are there fewer children than story characters?* (Yes.)

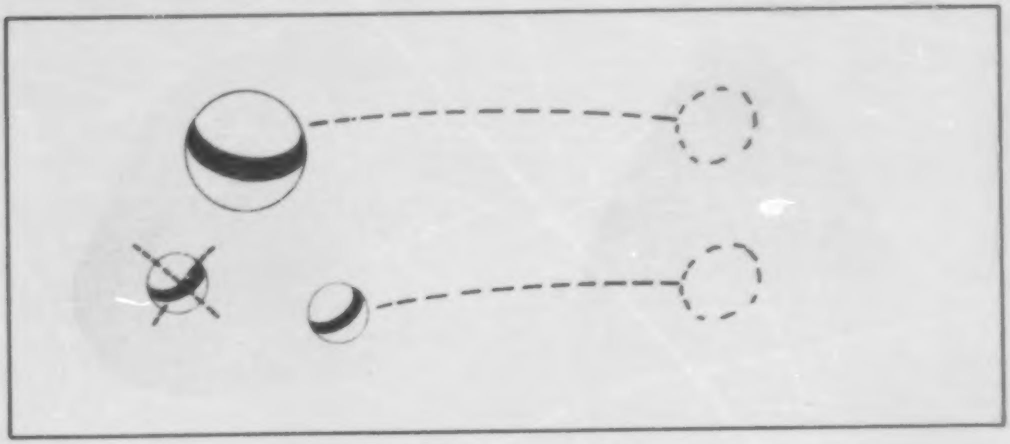
Ask the three children to put the pictures back on the flannel board and have another child give the pictures to the three children. Encourage this child to pair the pictures and the children in a different order.

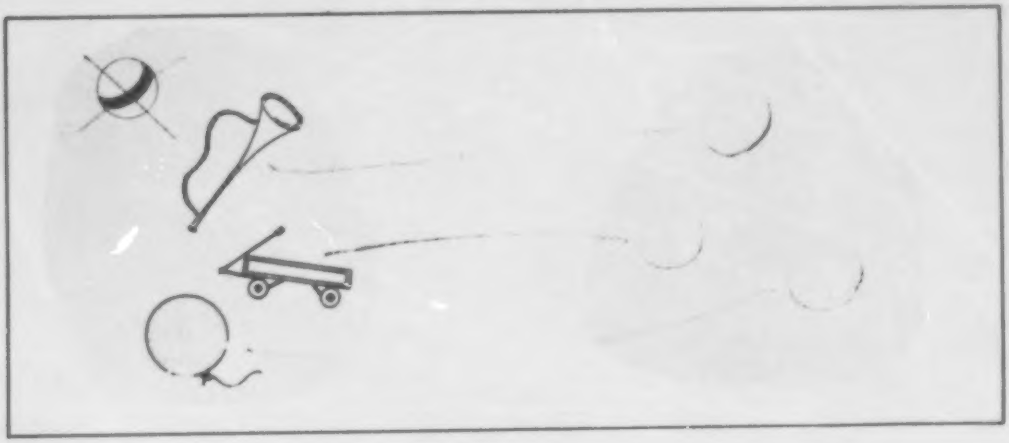
> *Do we now have as many children as story book characters?* (No.)
> *Are there fewer children than story characters?* (Yes.)
> *How many more story characters than children do we have?* (One.)

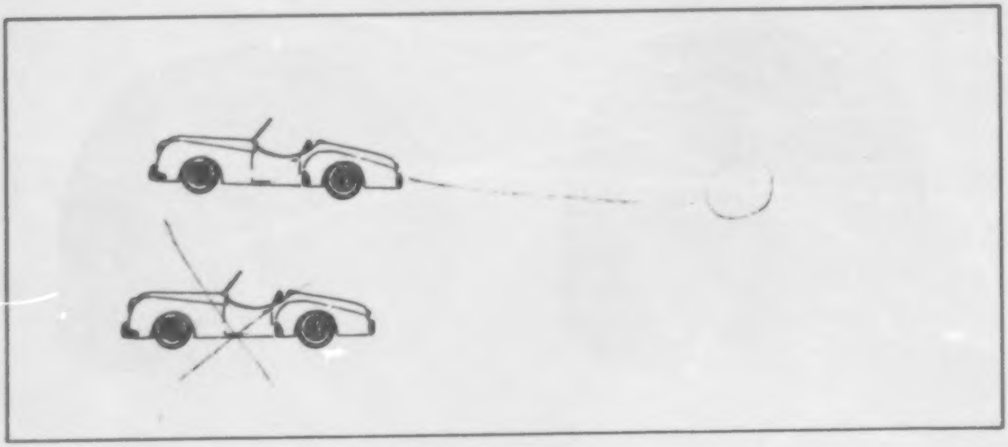
2. Choose some set of objects in the room and ask the children to look around for sets that are equivalent, for sets that have more members, and for sets that have fewer members.

3. Ask each child to fold a piece of paper, and to draw a set on one half of the paper. Then ask each to pass his paper to another pupil. That pupil is asked to draw a set with more (or fewer) members.

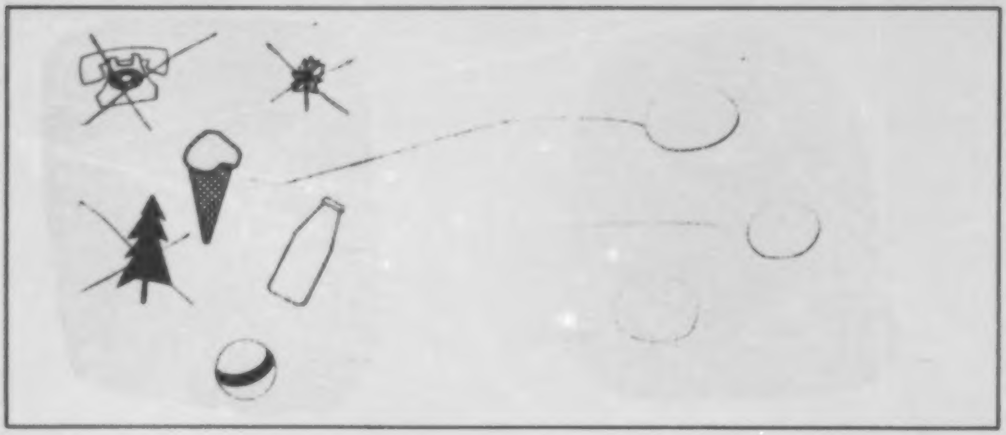
Show a set with fewer members

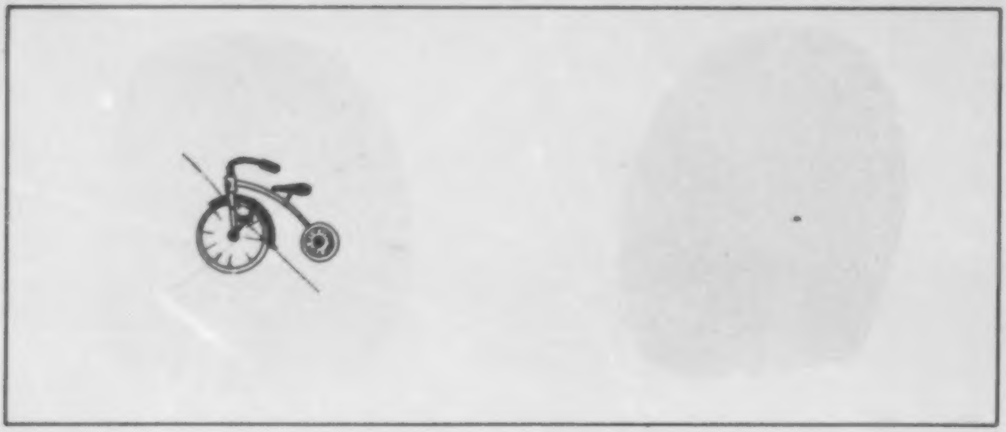


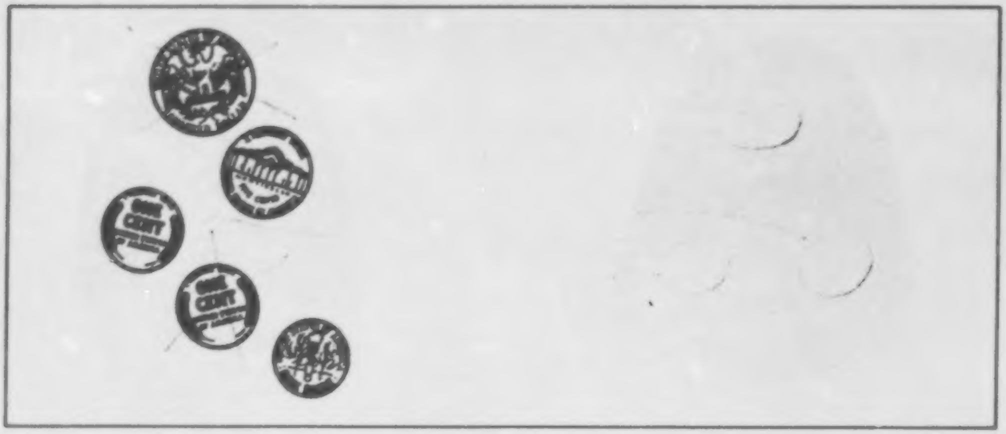




Show a set with fewer members






ONE CENT
ONE CENT

1-6. REVIEW

OBJECTIVES: To review the concepts of pairing, equivalent, more than and fewer than.

VOCABULARY: (No new words.)

MATERIALS: Pencils, crayons, or small objects.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Your children do not need this lesson if they understand clearly that they can always decide whether one set is equivalent to another, and whether one set has more or fewer members than another, by pairing members. If the children compare sets by counting, accept their answers but continue until they are sure that the comparison can be made without counting.

On a desk by the door, place as many pencils as there are boys in the class and as many crayons as there are girls in the class. As the children enter the room, ask each boy to take a pencil and put it on his desk and each girl to take a crayon and put it on her desk. Say:

> We discovered the other day when we paired our set of name cards with the set of children in our room that we had just as many members in one set as we had in the other set.
>
> Do you remember what we said about these sets? (Yes: equivalent.)
> We also discovered that two sets aren't always equivalent.
> We know that a set may have more members than another set or fewer members than that set.
> Let's check today to see if our set of boys is equivalent to the set of girls.
> If the two sets are not equivalent, then we can say that the set of boys has more or fewer members than the set of girls.

Have the children pair the members of the two sets (pencils and crayons) by asking a boy and a girl to go to the teacher's desk together and place their materials side by side. Continue until all children have gone to the desk. In case there are more boys or more girls, of course, they will not be able to go in pairs.

We have said that sets are equivalent when there are just as many members in one set as there are in the other.
Do we have equivalent sets of boys and girls?
Is the set of pencils equivalent to the set of boys?
Is the set of crayons equivalent to the set of girls?
Is the set of pencils equivalent of pencils equivalent to the set of crayons?

If the answer is "No," continue with these questions.

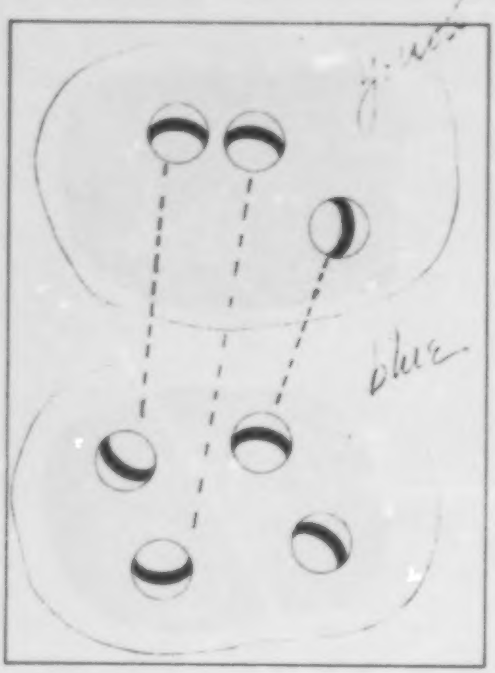
Can we say that the set of pencils has more members than the set of crayons?
Which set has more members?
How can you tell?
Can we say that one set has fewer members than the other set?
Which set has fewer members?
How can you tell?

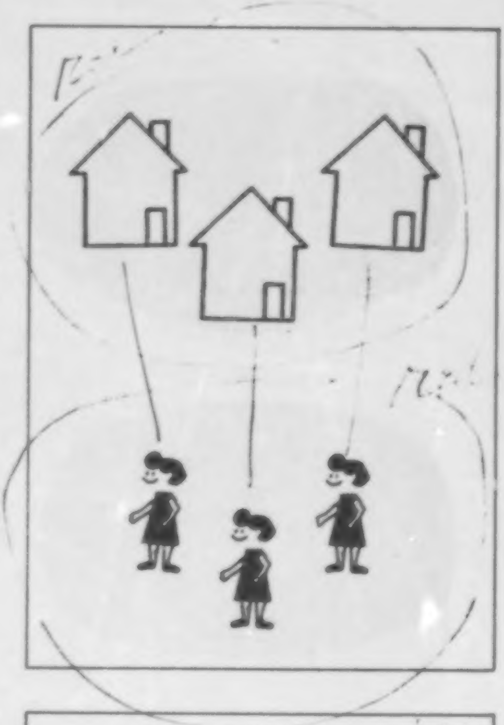
USING THE PUPIL'S BOOK, pages 40-41:

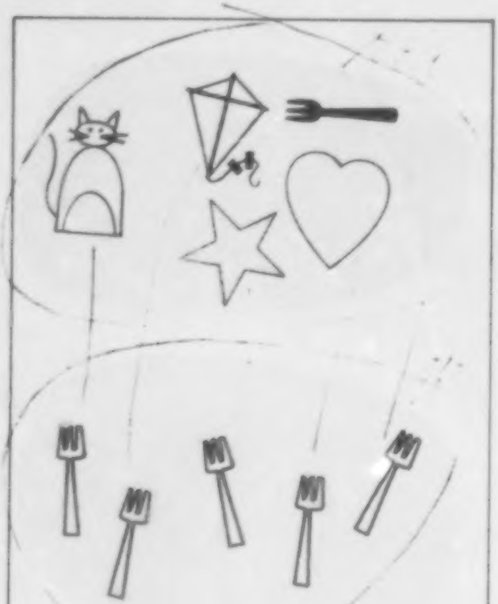
On Page 40, ask the children to pair the members of one set with the members of the other in each exercise. After they have completed one page, ask them to draw a red ring around those sets that are equivalent. For those sets that do not match, draw a blue ring around the set with more members and a yellow ring around the set with fewer members.

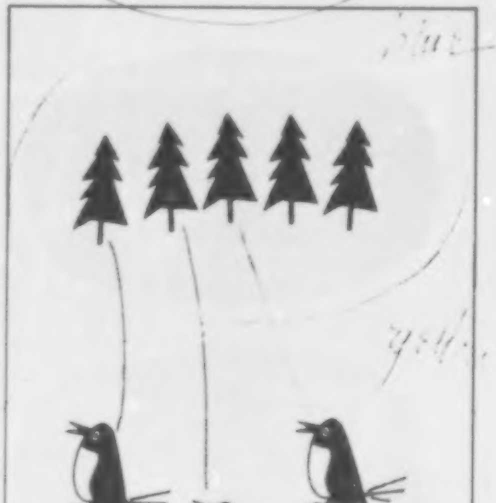
Then ask them to pair the members of the sets in each exercise on Page 41. After they have completed the page, give them instructions as above for identifying the equivalent sets and the sets with more than--fewer than members.

Compare these sets

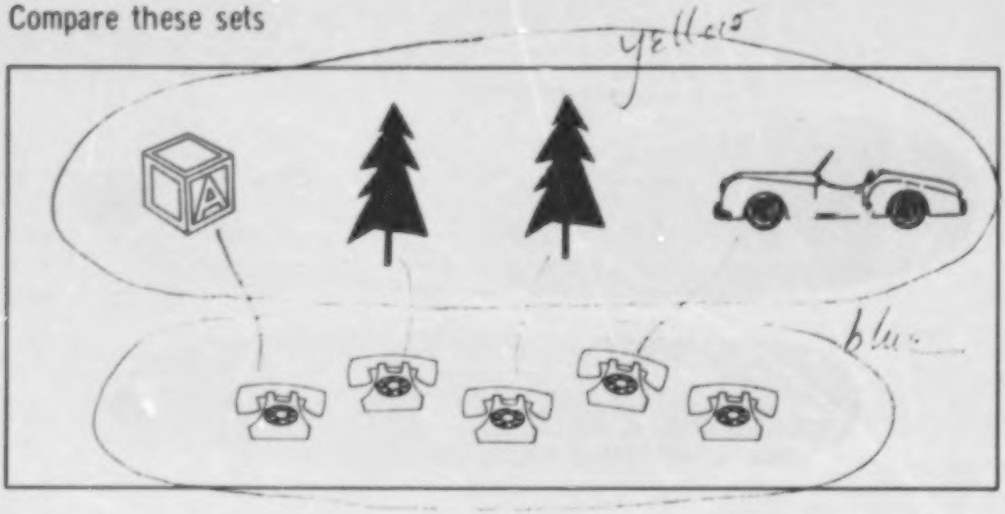


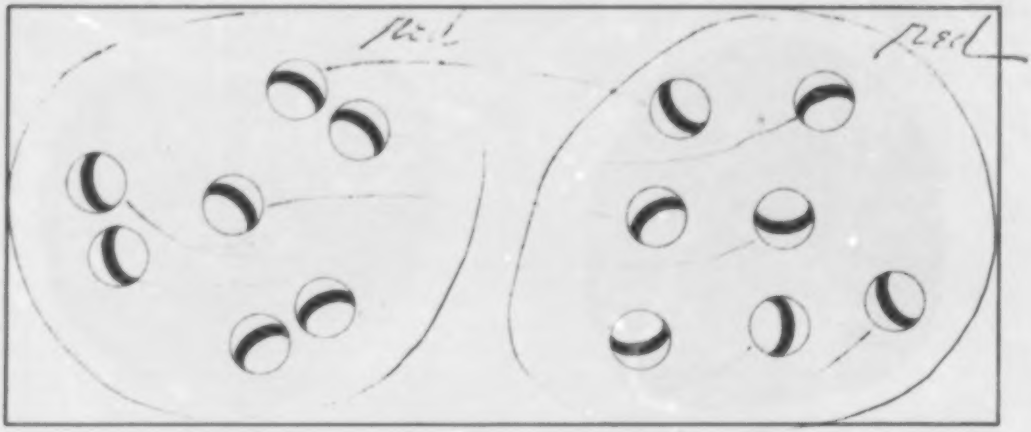


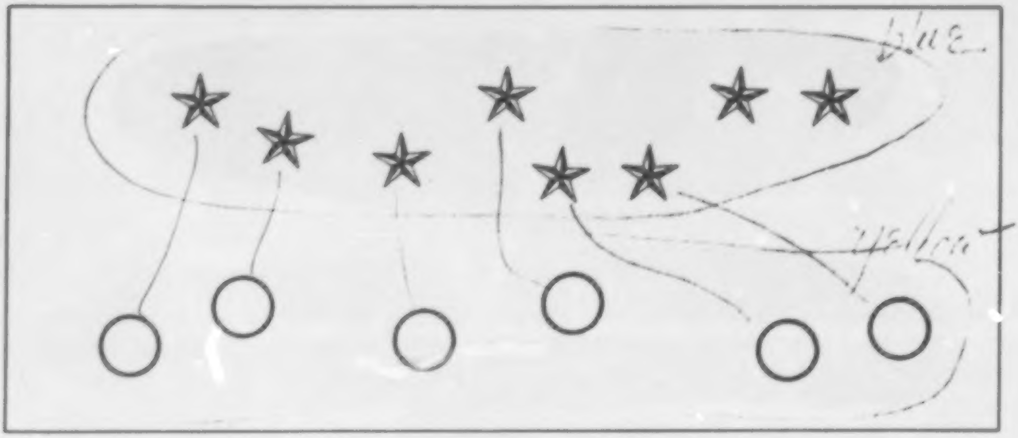



blue
yellow

Compare these sets


yellow
blue

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1-7. NUMBERS AND EQUIVALENCE

OBJECTIVES: To introduce the idea that equivalent sets have the same number of members, and that sets having the same number of members are equivalent.
To identify the number property of sets of 1-5 members.

VOCABULARY: Number, one, two, three, four, five.

MATERIALS: A number of dissimilar set objects for the teacher. These may be taken from sets of farm animals, fruits, and/or objects found in the classroom.

Sets of small objects such as blocks, buttons, bottle caps, disks, etc. for each child.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Hold a set consisting of models of an apple and a horse in one hand and another set such as a marble and scissors in the other. Ask the children to name the members of the sets. After some discussion of how the objects in the sets differ (An apple is a fruit. You can eat it. A horse is an animal. You can ride it. We don't ordinarily eat horse meat. A marble is a toy. You can play with it. A scissors is used to cut paper. We use a scissors at home and at school.), ask how the members of these sets are alike. The comments of the children should bring out the fact that the objects in the sets are not related in any usual way. Hopefully, someone will suggest that the two sets are alike in that both have the same *number* of members. If this does not happen, ask the following questions.

Are these sets equivalent? (Yes.)
What does equivalent mean? (The sets match. The members in the sets can be paired and there are none left over. Each set has as many members as the other.)
Yes, each of these sets has as many members as the other.
When a set has as many members as another, we say that the sets have the same NUMBER of members.
Do you know the name of the number that tells how many members these sets have? (Two.)
Are these sets equivalent? (Yes.)
Do these equivalent sets have the same number of members? (Yes.)

Remove the model of the apple from the first set.

> Are these sets equivalent? (No.)
> How do you know that is true? (There aren't as many members in the first set. The sets don't have the same number of members.)
> Very good. These sets do not have the same number of members.
> Do you know how many members are in the first set? (Yes, one.)
> In the second set? (Two.)
> These sets are not equivalent since they do not have the same number of members. Which set has more members? (Second set.)
> How many more? (One.)

Repeat the same procedure using two sets of 4, then 3 and 5. In each case remove a member from one of the sets after the children have established that the sets have the same number and what the name of that number is. This emphasizes the point that equivalent sets have the same number and sets that are not equivalent do not have the same number.

Provide each child with a set of objects. Ask them to construct a set with 1-5 members. Vary the request so that the sets are not ordered, for example, sets having 2, 4, 1, 3, 5 members. If the children hesitate, place an appropriate set of cut-outs on the flannel board and ask if their set is equivalent or has the same number of members as yours. Many such activities should be given the children so that they will learn to associate the number name with the correct number of objects.

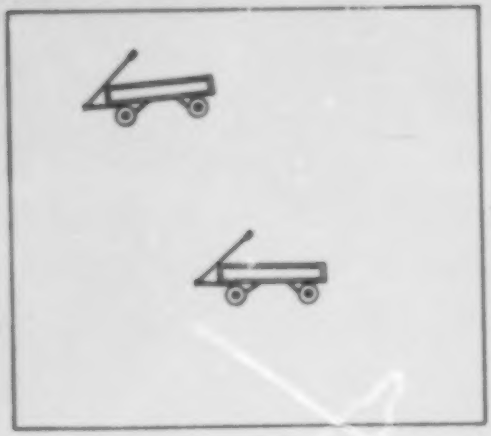
USING THE PUPIL'S BOOK, pages 42-43:

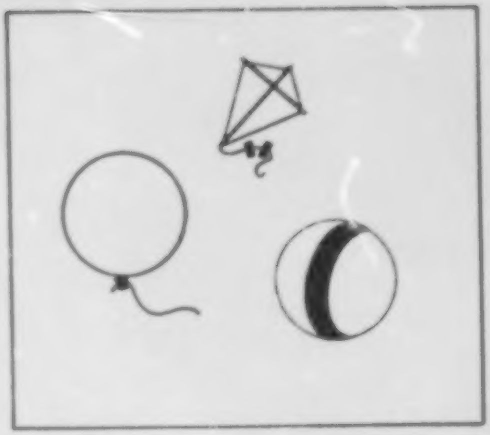
Use these pages to play a game. Give each child five blocks. Place a model set on the flannel board or display a set of objects. Ask a child to name the number of members in the model set. After the number has been named, direct the children to find all sets on the facing pages (42-43) of their book which have the same number of members as the model set. Direct them to cover these sets with blocks. Continue in a similar way with other model sets.

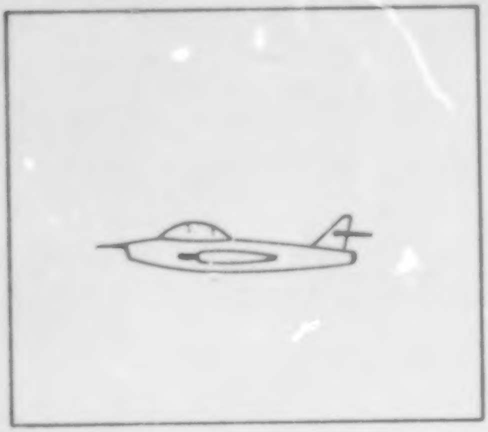
Pages 44-45:

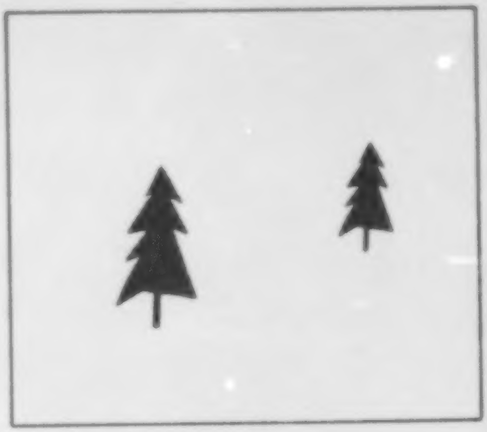
Display these pages so that all the children can see them. Give a number, for example, three. Ask a child to come and point to all the sets which have three members. Have other children do the same for the sets in their books. Continue with 1, 2, 4, and 5.

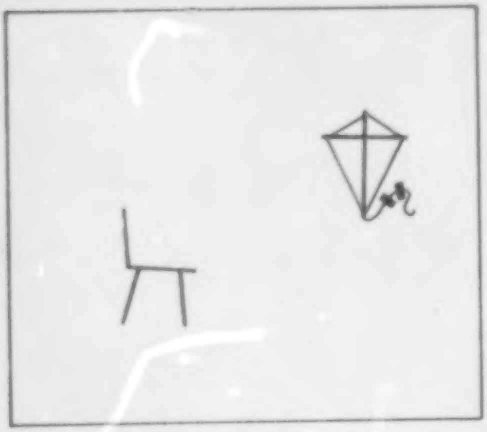
Same number of members

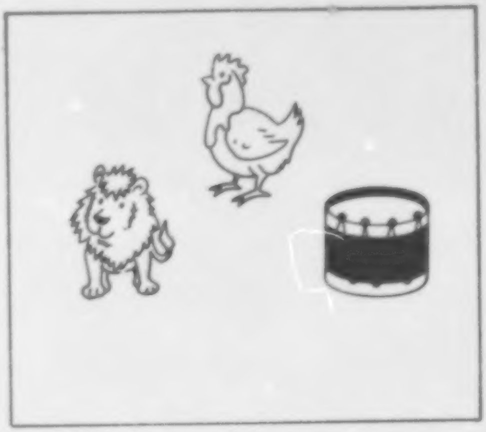




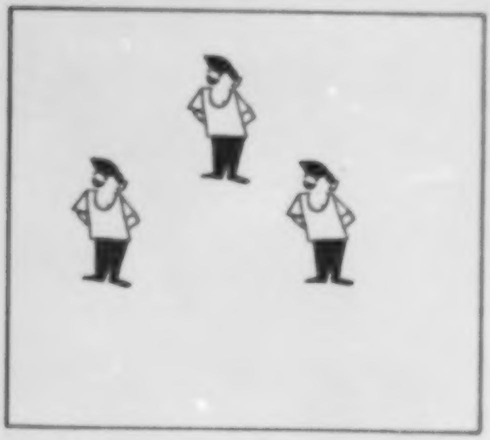


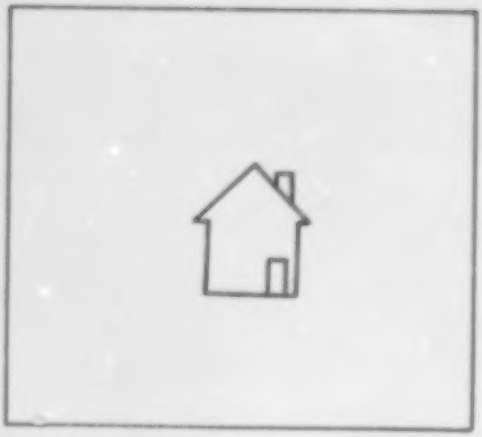


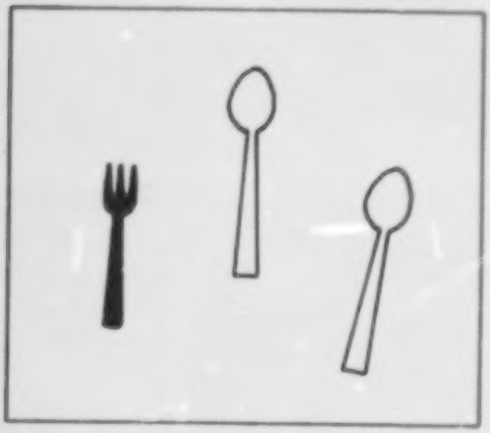


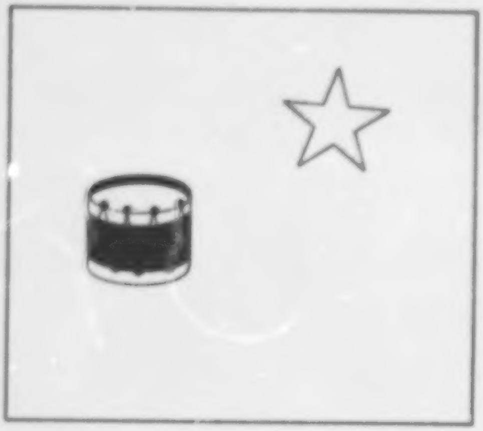


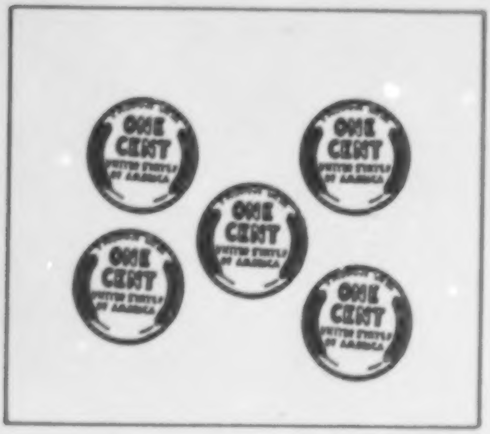
Same number of members







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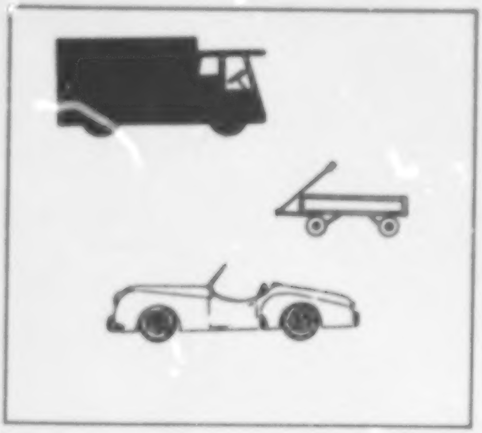
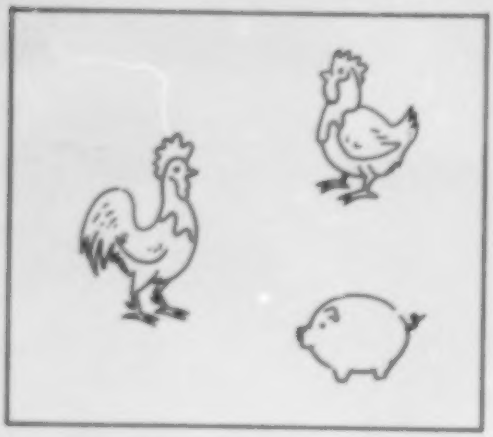
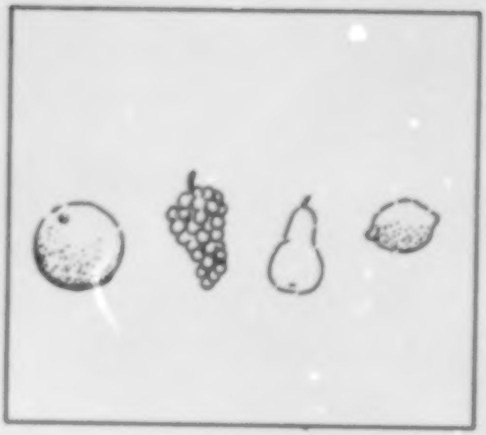
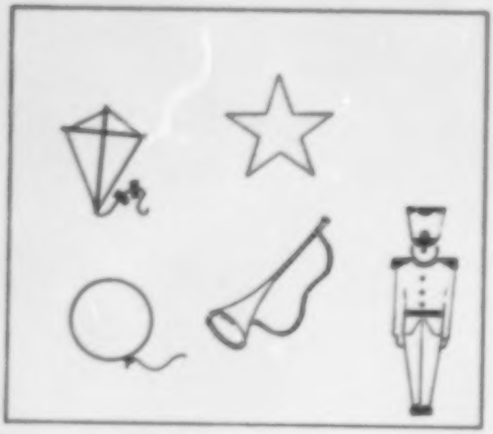
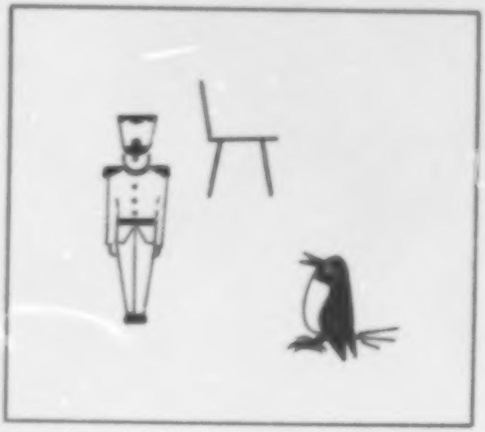


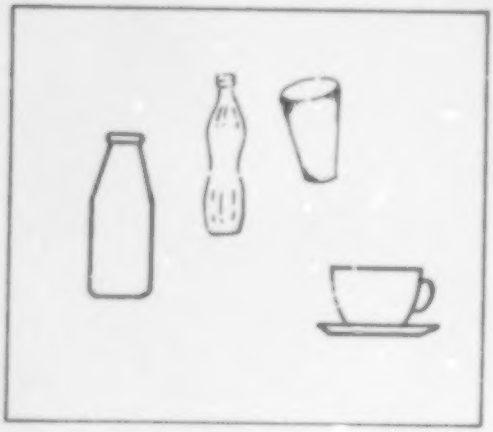
Same number of members

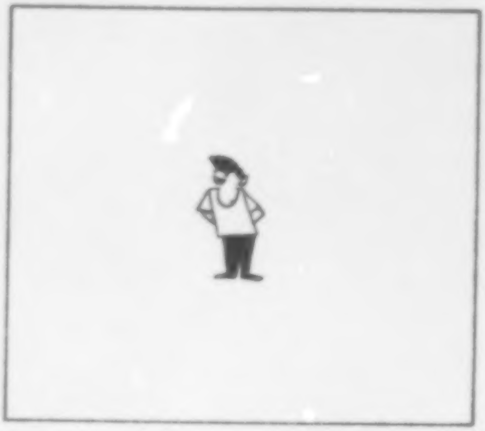












1-8. NUMBER PERCEPTION, WITHOUT COUNTING

OBJECTIVES: To help children perceive, without counting, the number of objects in a set of no more than five members.

To develop the understanding that zero is the number of members of the empty set.

VOCABULARY: Zero.

MATERIALS: A set of five objects; e.g., pencil, crayons, eraser. Perception cards, in various patterns, sets zero through five; set objects for the children.

SUGGESTED PROCEDURE

PRE-BOOK ACTIVITIES:

Place the set of objects on a demonstration table. Have children grouped around where all can see. Question children concerning the number of the set (five). Ask a child to come up and take an object to his chair. Repeat, each time asking for the number of the set that remains. When the last object is removed, ask about the set remaining on the table.

What set do we have now on the table? (The empty set.)
Do you know a number that tells how many things there are in the empty set? (Zero.)
Zero is the number that tells us how many things there are in the empty set.

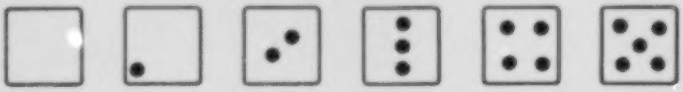
Continue with illustrative questions relative to the children's experience, e.g.:

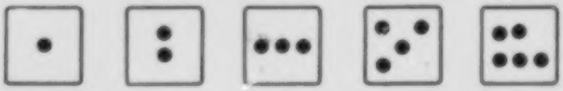
Mary had two balloons.
Bill broke both of them.
What number tells us how many balloons Mary had left? (Zero.)

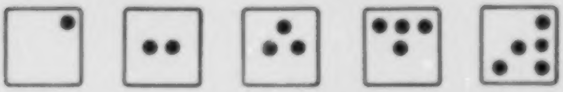
Joe had a piece of candy.
He ate it at lunch time.
What number tells us how many pieces of candy he has now? (Zero.)

- Count a set of objects and put them in a grocery bag. Remove one object at a time and ask a child to tell the number of objects still in the bag. Continue until there are no objects in the bag. Reinforce the idea that zero is a number.

Use perception cards (without numerals) of sets of from zero through five members. Perception cards should have different arrangements of the members of the sets:







Give each child five objects. Show a card and have the child put on his desk a set equivalent to that displayed on the card.

Put a set of objects on the flannel board. Have children show equivalent sets at their desks.

Display a perception card very quickly; then conceal it and have children show equivalent sets at their desks.

Display a card, cover it, and have a child give orally the number of the set that was shown.

FURTHER ACTIVITIES:

There are many opportunities to point out equivalent sets in the classroom and to encourage children to perceive the number of the set without counting. For instance, if there are two sides of an easel, two children can paint. If there are four balls for the recess period, four children may take out balls. If there are three fish in a fish bowl, put three books about fish on the table beside them and encourage children to see "three," rather than "one, two, three."

1-9. COMPARISON OF NUMBERS

OBJECTIVE: To introduce the concepts of greater than, less than.

VOCABULARY: Greater than, less than.

BACKGROUND NOTE:

To compare two numbers, say 5 and 3, we choose a set of 5 members and a set of 3 members and by pairing find that the first set has more members than the second. We therefore say that 5 is greater than 3 and that 3 is less than 5. We use the terms "more than" and "fewer than" in connection with sets, and "greater than" and "less than" in connection with numbers.

MATERIALS: Flannel board and cut-outs for the teacher; sets of small objects for the children.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

You will want to introduce the concept of "greater than" first, perhaps waiting several days before considering "less than." Children find the latter much more difficult to grasp. It is essential that you use the correct terms while permitting the children to use primitive expressions which indicate that they understand the concept. For example, when asked why 4 is greater than 2, many children will answer, "There are more" or "More." This is acceptable since it implies that the child does understand the idea. The teacher, however, should respond with, "Yes, that is correct. Four is greater than 2 because a set of 4 has more members than a set of 2." Constant reinforcement of this kind which encourages the child but does not impose a formality of language for which he is not ready will deepen understanding and eventually result in the development of the precise language. There is no need to drill on this language.

Have the children build sets with 2 and 3 members. Ask the children to put the sets at the front of their desks.

> Which set has more members? (The set of 3 members.)
> Why do we say this set of 3 members has more members? (If we pair members of this set with those of the set of 2, we have a member left over.)

Since a set of 3 things has more members than a set of 2 things, we say that the number 3 is GREATER THAN the number 2.

Repeat with several other examples.

Ask the children to think of numbers greater than 2. Do not limit them to the number 9 but insist that each child explain why his number is greater. (It is the number of a set with more members than 2.)

- Have the children use small objects to dramatize the solving of word problems. (These are supposed to be both vocabulary work and mathematics.) In the first problem, a technique for solving such word problems is suggested within the parentheses.

1. Johnny has 3 marbles. (Use your blocks to represent the marbles Johnny has.) Jim has 5 marbles. (Make another set of blocks to represent Jim's marbles.) Which boy has the greater number of marbles? (Pair the members of the sets.) Which boy would have some marbles left over?

   Use the same technique with these word problems.

2. Mary has 2 balloons. June has a greater number of balloons. How many balloons does June have? (Any number greater than 2.)

3. Joe has 4 candy bars. Jim has 5. Which one has the greater number of candy bars?

4. Elizabeth has 3 sticks of gum. Rosa has none. Does Rosa have the greater number of sticks of gum?

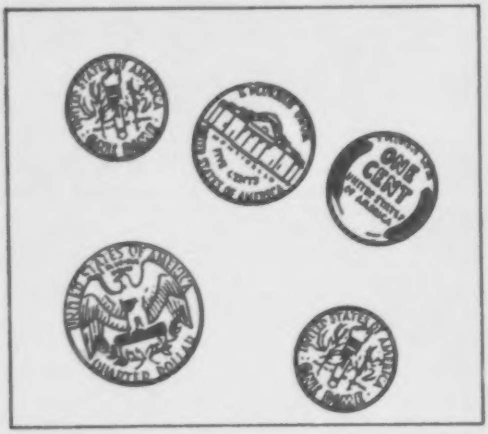
5. The boys have 5 balls. The girls have 5 dolls. Do the boys have the greater number of toys?

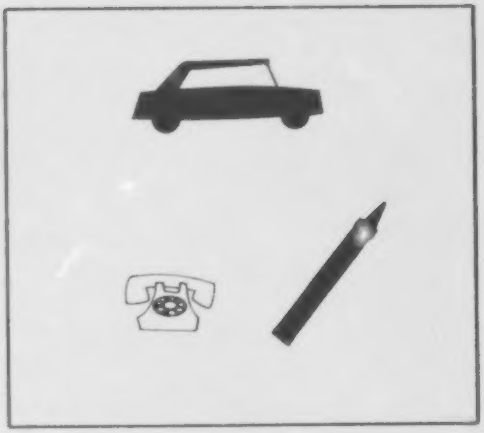
Have the children build sets with 2 and 4 members. Ask the children to put the sets at the front of their desks.

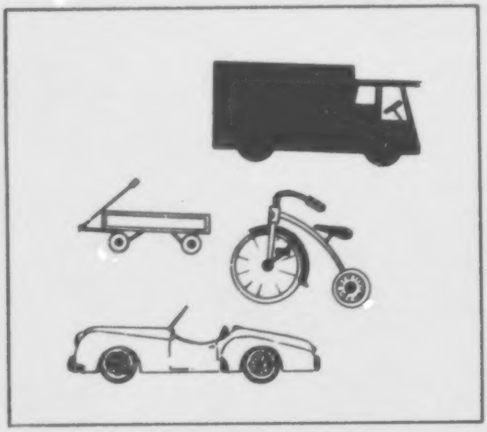
> Which set has fewer members? (The set of 2 members.)
> Why do we say the set of 2 members has fewer members? (If we pair members of this set with those of the set of 2, we have members left over. The other set has more.)
> Since a set of 2 things has fewer members than a set of 4, we say that the number 2 is less than the number 4.
> Is 2 less than 4? (Yes.)

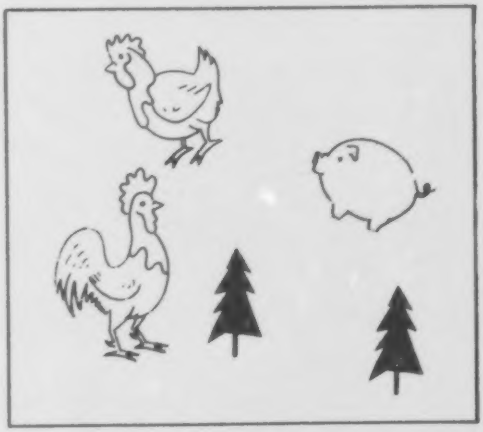
Repeat with several other examples.

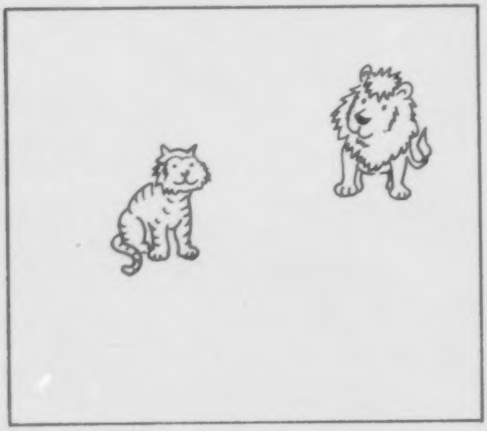
Same number of members

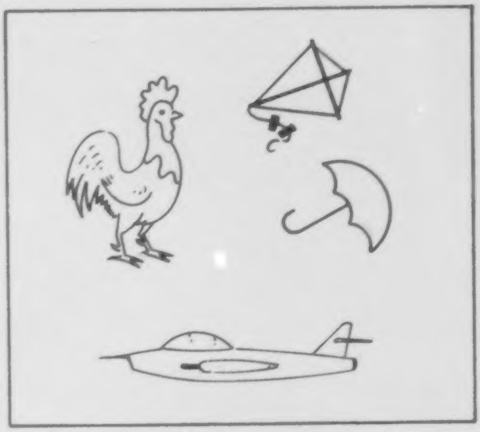












- Ask the children to think of a number less than 5. As they give their answers, ask why the number is less. Repeat with other numbers.
- Have the children use small objects to dramatize the solving of word problems. In the first problem given below, a technique for using such word problems is suggested within the parentheses.

1. Sue has four dolls. (Use your blocks to represent the dolls.) Mary has 3 dolls. (Make another set of blocks to represent Mary's dolls.) Is 3 less than 4? How did you know?
2. Jim has 3 balloons. Bob has only 1. Would you rather be Jim or Bob? Why? Is 1 less than 3?
3. Bob has 4 balls. Jim has less than 4. How many did Jim have? (0, 1, 2, or 3.)
4. Bill made 3 mistakes on his reading paper. Joe made 5 errors. Bill said that he was glad he wasn't Joe. Why did he say that? (Bill made fewer errors. 3 is less than 5.)

Many children will need a great many experiences using blocks or other set objects in showing that their answers to such questions as those below are correct.

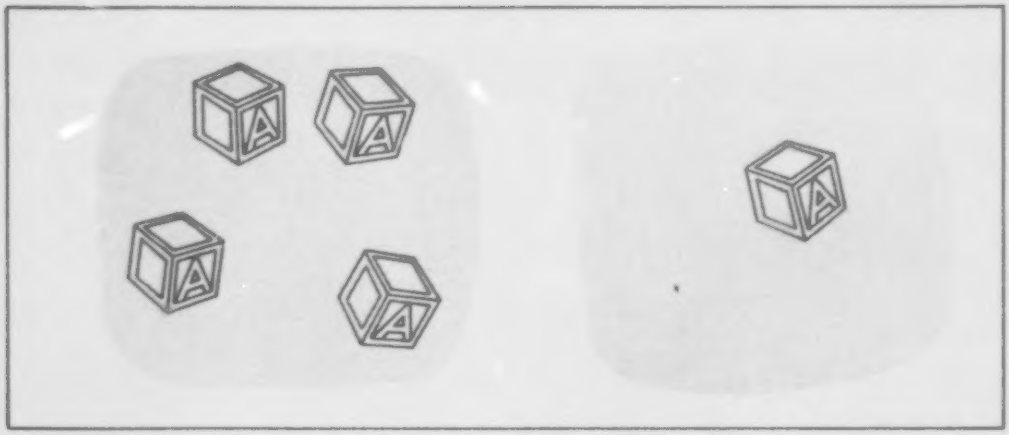
Is 4 greater than 3?
Is 2 less than 5?
Is 1 less than 3?
Is 5 greater than 4?

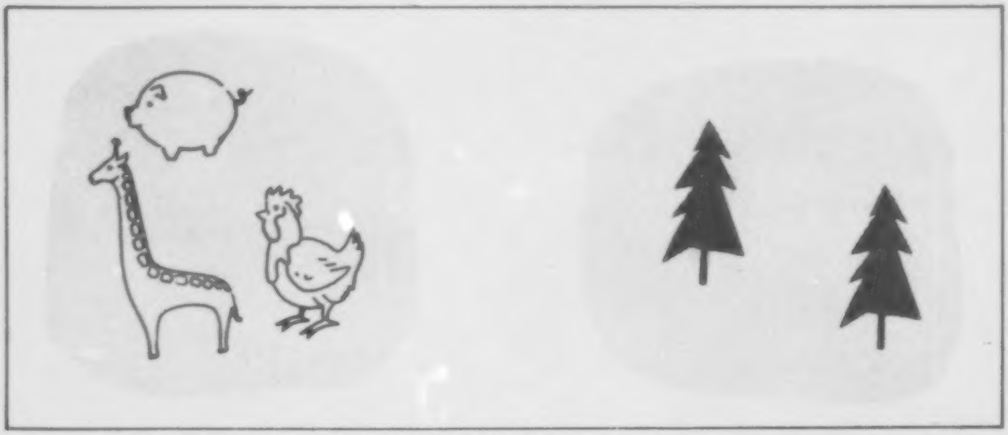
USING THE PUPIL'S BOOK, pages 46-47:

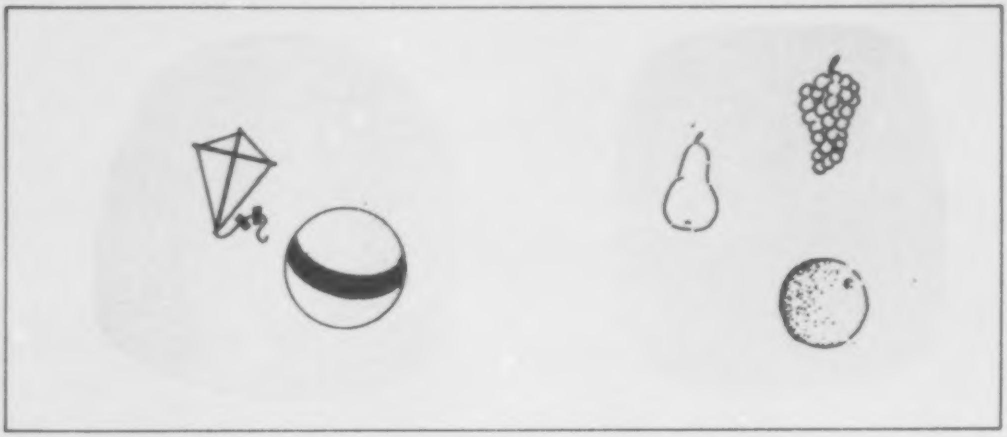
For each exercise, ask the children the number of each set. Then ask which number is greater, for example, 4 or 1. Have each child put a block (or some other kind of marker) on the set which has the greater number.

These pages may be used again in your discussion of which number is less than, asking children to place a marker on the set which has the lesser number.

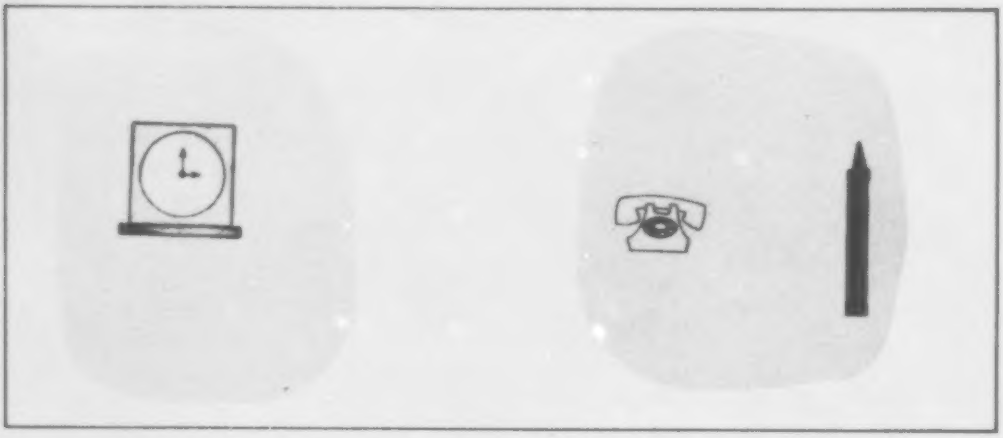
Greater than - less than

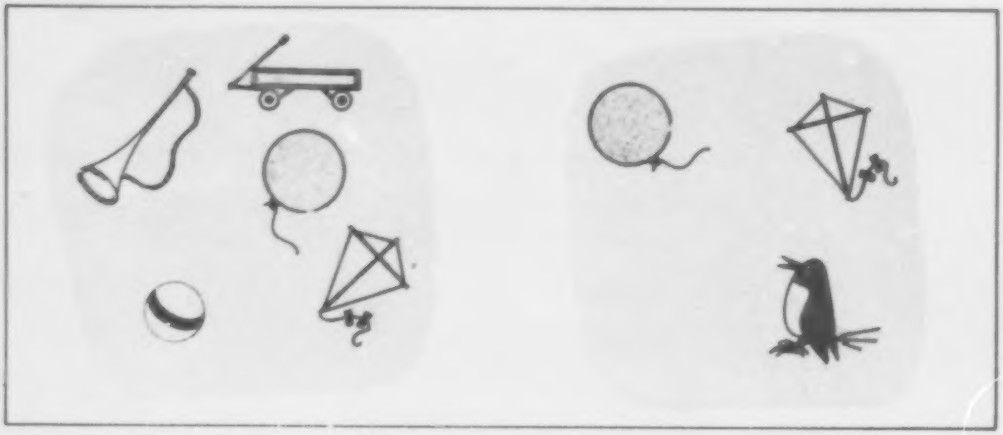


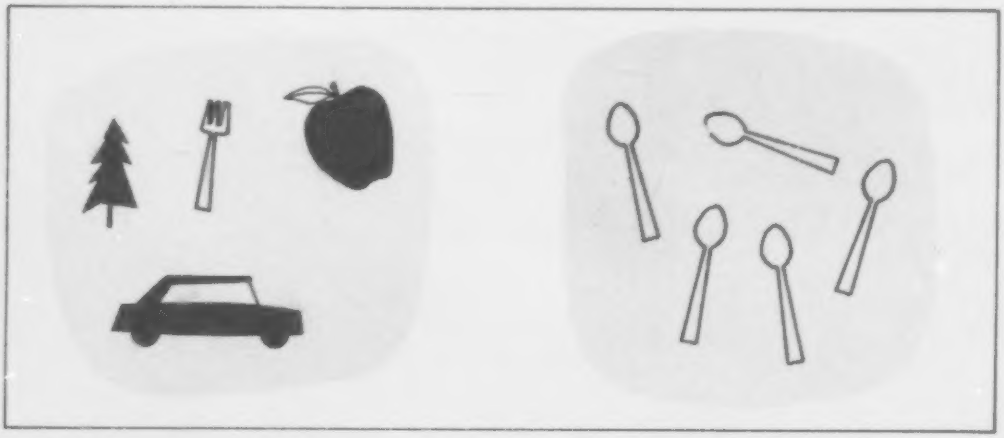




Greater than - less than







Chapter 2

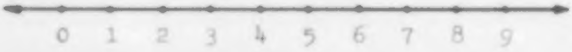
NUMERALS AND THE NUMBER LINE

BACKGROUND

This chapter has several objectives. First, we wish to teach children to recognize and to write the numerals: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each of these symbols must be connected with the correct spoken word and with those sets having the correct number of members. It is worth noticing that children in the first and second grades have to establish the association of four different sorts of objects with each number. For example, they must learn to connect the number eight with sets having eight members, with the symbol "8", with the written word "eight", and with the spoken word "āte". At the same time children are learning other names for 8, such as 4 + 4, 9 - 1, and so on. Each of these associations must be established by many different experiences.

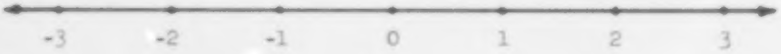
Second, we wish to reinforce the connection between comparison of sets and comparison of numbers. We recall that one set has fewer members than another if, when the members of the first set are paired with the members of the second, there are members of the latter left over. This relationship between sets leads to a relationship between numbers. We say that 5 is greater than 3, and that 3 is less than 5 because if the members of a set of 3 are paired with the members of a set of 5, there will be members of the latter set left over. Recall that if one set has more members than another, then the number of members of the first set is greater than the number of members of the second set; if the first set has fewer members, then the number of members of the first set is less than that of the second; and if sets are equivalent then they have the same number of members. The words "more than", "fewer than", and "equivalent" refer to sets. The words "greater than", "less than", and "equal" refer to numbers.

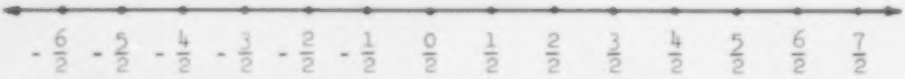
Lastly, this chapter begins the development of the concept of the number line. At this stage, we are trying to help children think of the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 as equally spaced labels on a line. We put arrows on the ends of the line we draw to help children understand that a line extends indefinitely in both directions, but the picture we want to give now is the following:

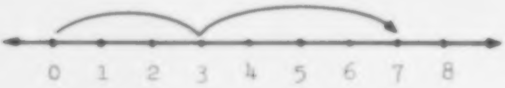

0
1
2
3
4
5
6
7
8
9

Be sure that they understand that the numerals are associated with the number of steps from 0. The "starting point" is 0, and each number is the number of "steps from the starting point".

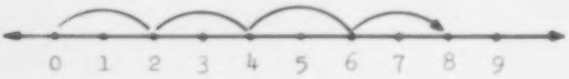
Later, pictures such as these will be used:




$-\frac{6}{2}$
$-\frac{5}{2}$
$-\frac{4}{2}$
$-\frac{3}{2}$
$-\frac{2}{2}$
$-\frac{1}{2}$
$\frac{0}{2}$
$\frac{1}{2}$
$\frac{2}{2}$
$\frac{3}{2}$
$\frac{4}{2}$
$\frac{5}{2}$
$\frac{6}{2}$
$\frac{7}{2}$



$3 + 4 = 7$


0
1
2
3
4
5
6
7
8
9

$4 \times 2 = 8$

Without going into details, these pictures indicate that the number line will be used to motivate the introduction of negative numbers, to emphasize that one number may have many names, to illustrate the notion of addition and its properties, and to give an interpretation of multiplication. We use the number line in this chapter primarily to reinforce understanding of the natural ordering of the numbers, but there are many other pedagogical uses for the concept. There are also profound mathematical reasons for the early introduction of the number line. We are associating an algebraic object (a number) with a geometric object (a point), and this foreshadows the close connection between algebra and geometry which was discovered by Descartes and called analytic geometry.

IMPORTANT NOTE:

Section 2 of this chapter requires that the children be able to write numerals. You should teach the writing of the numerals in the writing period but not until children have the understandings and abilities developed by the end of Section 2-2 Part I for the numeral 0 - 5 and Section 2-2 Part II for the numerals 6 - 9. It can not be assumed that if children can recognize the numerals, they can write them. They will need a great deal of careful practice which can be given independent of the mathematics lesson.

2-1. ARRANGING SETS IN SUCCESSION

OBJECTIVE: To put sets with 0 through 10 members in order so that each non-empty set has one more member than the set which precedes it.

VOCABULARY: (Review) order, more than, fewer than.

MATERIALS: Materials to be used on flannel board, including numerals and pairing symbols or pieces of string (or similar materials for the bulletin board or the magnetic board); sets of objects for children.

SUGGESTED PROCEDURE:

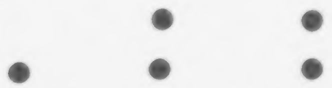
PRE-BOOK ACTIVITY:

The children have had many experiences with sets of 1 - 5 members. Consequently, it seems wise to order such sets first and then direct attention to the empty set as the set preceding the set of one member. A considerable amount of time will be needed to develop the number property of sets of 6 - 10 members and the names of these numbers. Counting, partial counting beginning with a subset (part of a set) of more than 1 member insofar as is possible, will then be used as a technique for determining the number property of a set of more than 5 members. This does not imply that many children will be able to recognize the number of a subset and proceed with the counting at this time, but efforts should be made to encourage such a procedure. Again, the experiences provided later in the curriculum will help develop this ability.

Place sets of 1 and 2 members on the flannel board.

> Are these sets equivalent? (No.)
> How can you tell just by looking at these sets that they are not equivalent? (They don't match. The second set has 1 more member. One set has 1 member; the other has 2.)
> Is 2 greater than 1? (Yes.)
> A set of 2 has more members than a set of 1.

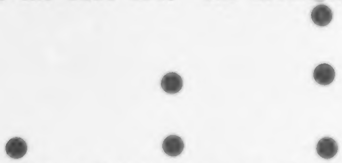
Place another set of 2 members on the flannel board so that the arrangement now looks like this:



Point to the two sets of 2 members.

> Are these sets equivalent? (Yes.)
>
> How do you know? (Both sets have the same number. Both sets have 2 members.)

Place another cut-out in the last set. The arrangement now looks like this:



Indicate that you are speaking about the sets of 2 and 3 objects.

> Are these sets equivalent? (No.)
>
> Why not? (They don't have the same number. The last set has more.)
>
> How many more? (1.)
>
> Is 3 greater than 2? (Yes.)
>
> A set of 3 has 1 more member than a set of 2.

Continue in the same way to order the sets of 4 and 5 members.

> Look carefully and see if you can tell me something about the way in which I have placed these sets on the flannel board.

If no one comments that each set has 1 more member than the set before it, ask such questions as:

> What is the number of the first set? (1.)
>
> The next set? (2.)
>
> Does a set of 2 have more members than a set of 1? (Yes.)
>
> How many more? (1.)
>
> What is the number of the next set? (3.)
>
> Does a set of 3 have more members than a set of 2? (Yes.)
>
> How many more? (1.)

If the desired response concerning the ordering is not obtained, proceed with similar questions regarding sets of 4 and 5. Then continue.

> Look at the first set.
>
> Does each set after the set of 1 have 1 more member than the set before it? (Yes.)
>
> When we have placed sets in this way, we say that we have ORDERED the sets.

What set would come before the set of 1 member? (The empty set.)

Do I need any cut-outs to represent the empty set? (No. The empty set doesn't have any members.)

What is the number of the empty set? (Zero.)

Point to each succeeding set and ask the children to tell you the number of the set. Then remove all cut-outs from the flannel board and replace with five sets of objects ranging from one to five members in the arrangement suggested below. Do not, however, use geometric shapes as set objects.

```
                  *
                  *    *
*                 *    *
*         *       *    *
*    *    *       *    *
```

Point to each set and ask the children the number of the set.

Are these sets arranged in order? (No.)

How do you know they aren't in order? (You should start with a set of 1. Each set should have one more.)

Which set has the fewest members? (Set of 1.)

Where (indicating with your hand) should I place the set of 1? Before the set of 3 members or after the set of 4 members?

Agree that the set with one member can be at the left and place it there. Next point to the set with three members.

Does this set (the set with three members) have more members than the set with one member? (Yes.)

Does the set of 3 members have 1 more member than the set of 1 member? (No, it has 2 more.)

The set of 2 members should come after the set of 1 member.

Place the set of 2 members between the set of 1 member and the set of 3 members. Bring out by pairing that the first three sets are in order since each set has one more member than the set before it. Complete the ordering using the techniques suggested above. Then ask if there is any set with fewer members than the set with one member. Emphasize that the empty set belongs on the left-hand side. (No picture, of course.)

USING THE PUPIL'S BOOK, pages 48-49

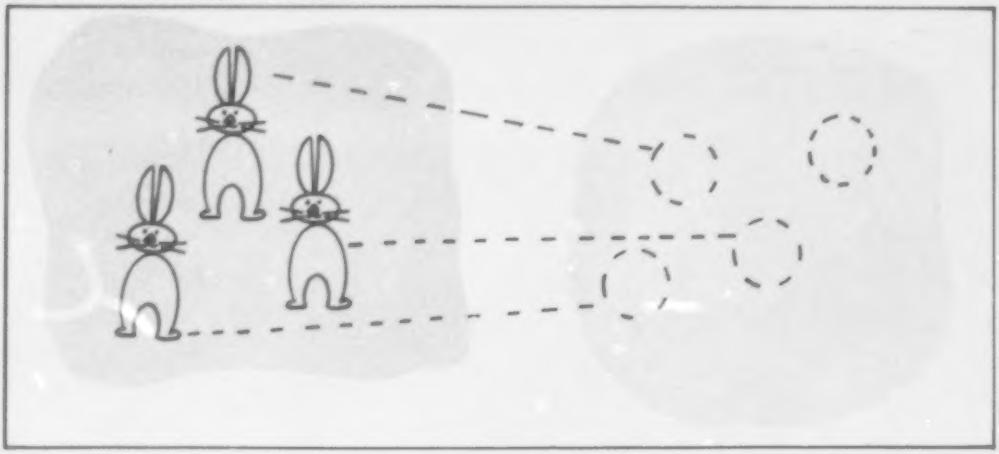
Pages 48 and 49 are teaching pages. Have the students turn to Page 48 of the Pupil's Book. Display your book so the children can see the page. Read the title of the page. Direct the pupils' attention to the set of rabbits at the top of the page. Ask the students to give the number of the set (three). Explain that they are to draw a set of rings in the space provided (point to the space) which has one more member than the given set. Suggest that the children represent an equivalent set first by drawing a line from a rabbit to the shaded region, then drawing a ring at the end of the line. Have them do this by tracing over the dotted marks. When the student has done this for every rabbit, then he has drawn a set of rings which is equivalent to the set of rabbits. To draw a set with one more member, he merely draws an extra ring. Help the children work each set on these two pages.

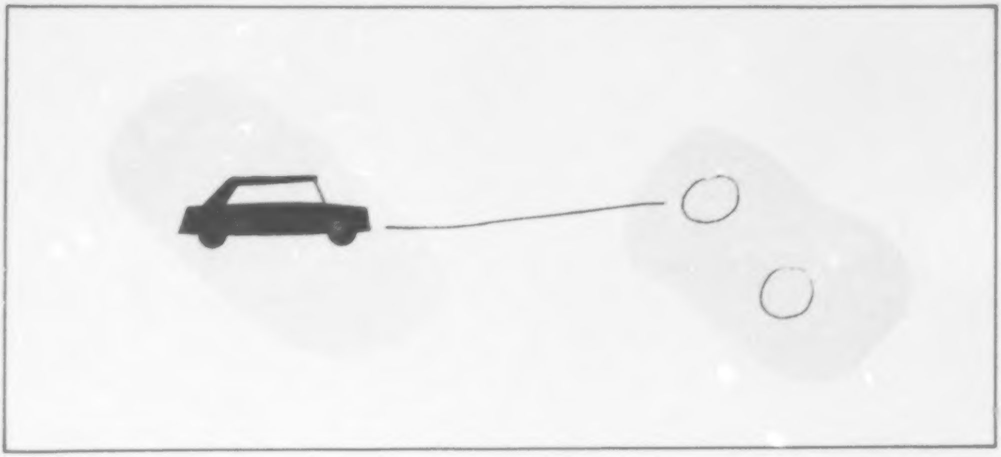
Note that the left frame of the bottom left corner of Page 49 represents the empty set, so the children should draw a set with one member in the right frame.

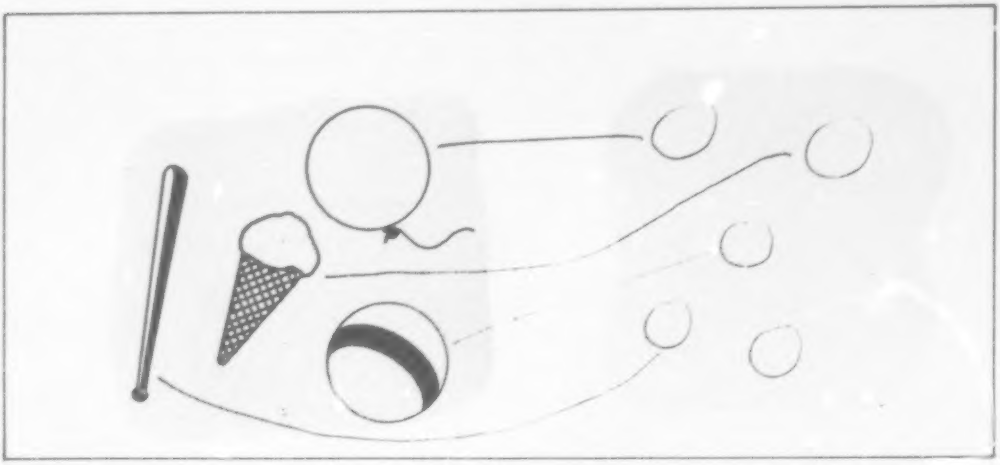
Pages 50 and 51 are for independent work.

Teaching Page

Sets with one more member

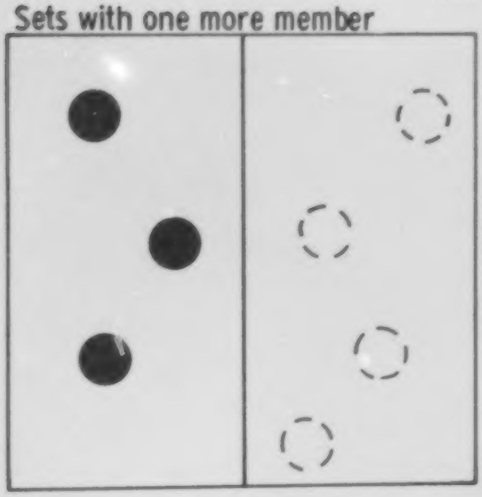


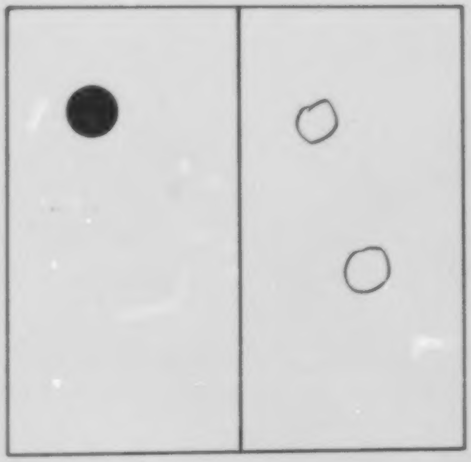


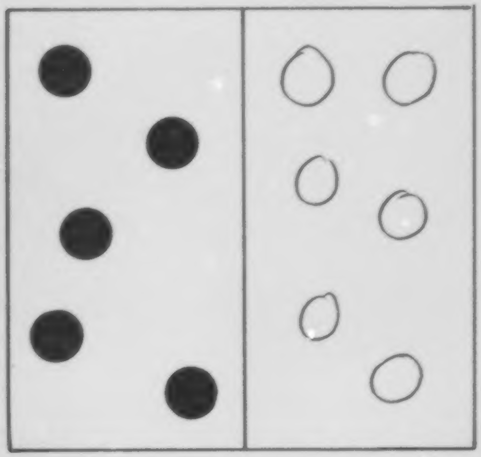


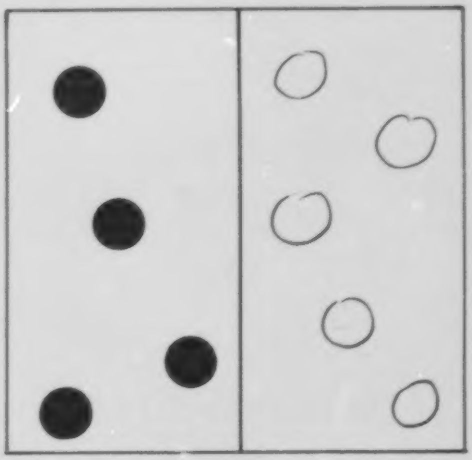
Teaching Page

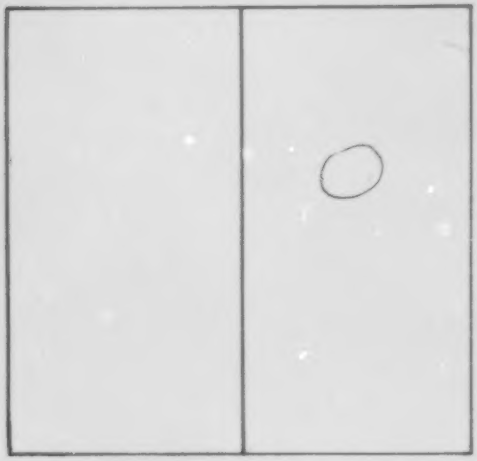
Sets with one more member

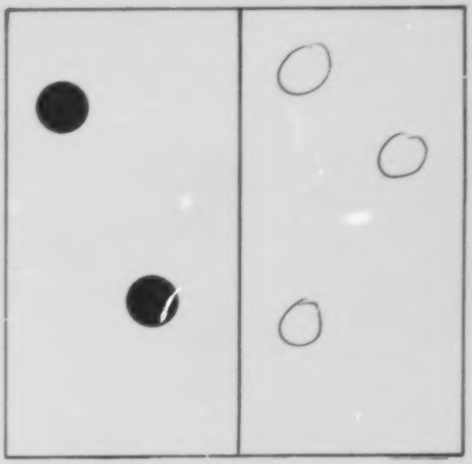




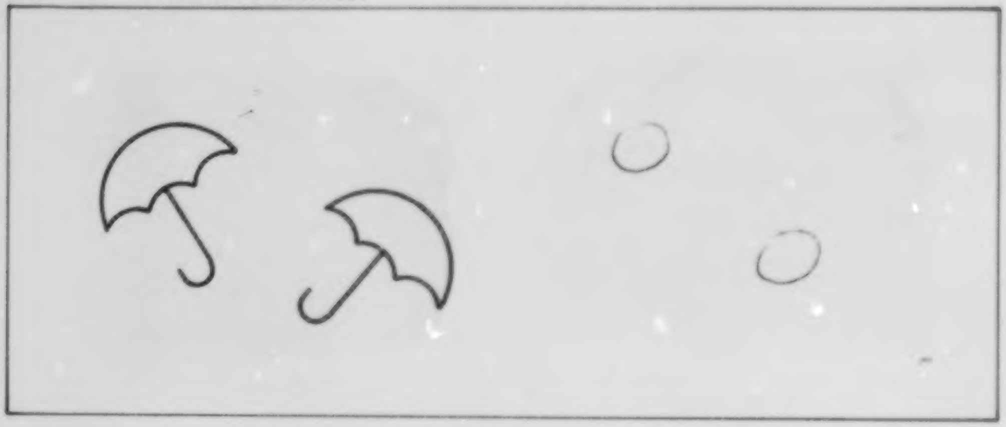


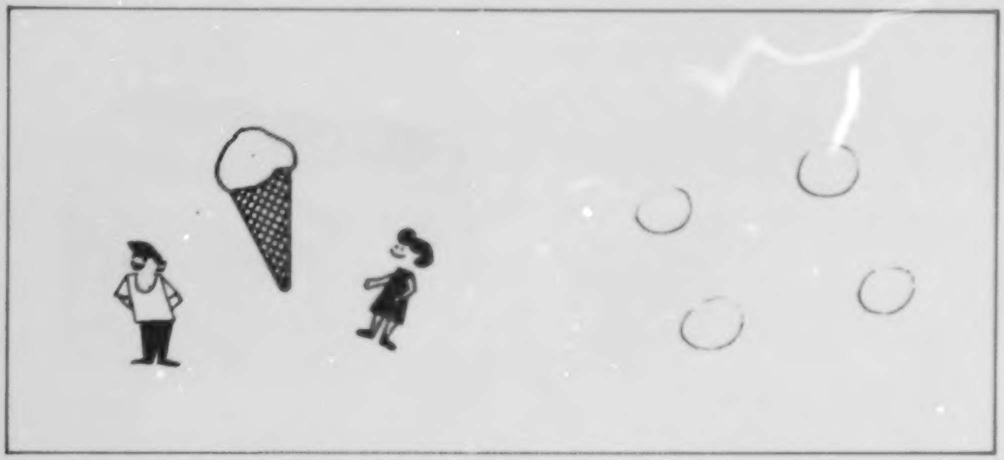




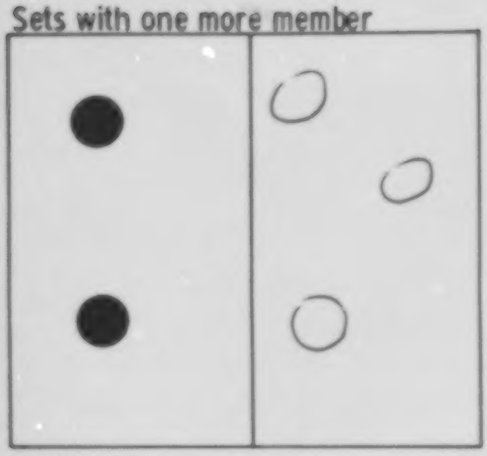


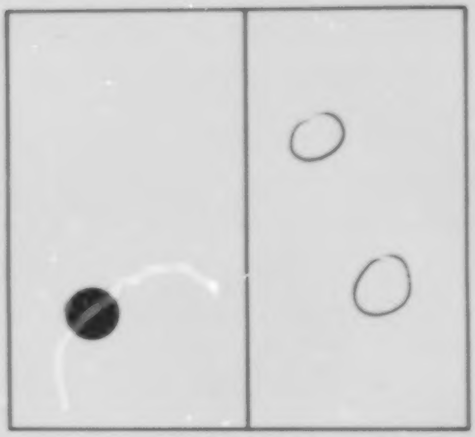
Sets with one more member

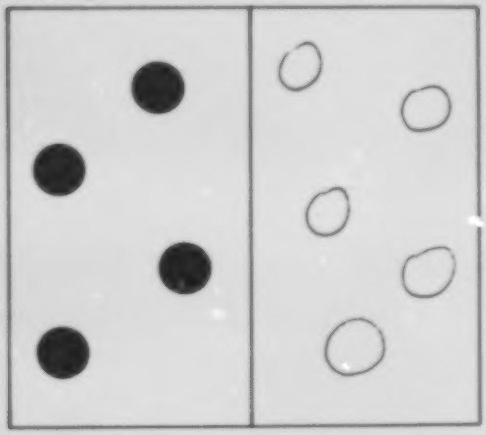


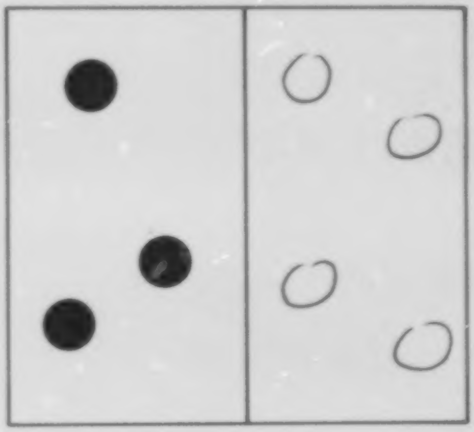


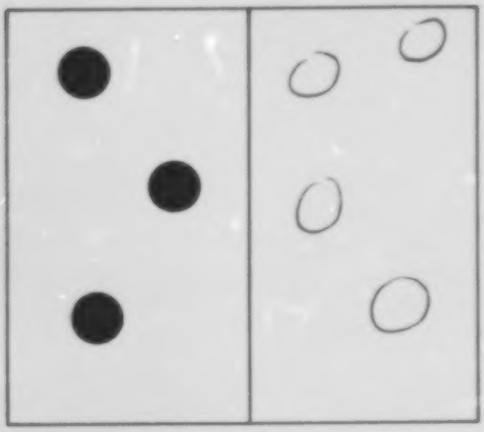
Sets with one more member

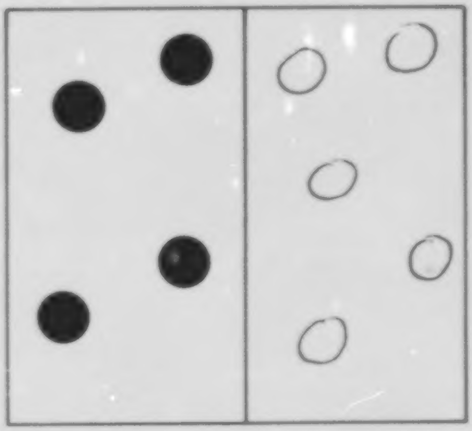












USING THE PUPIL'S BOOK, pages 52-53

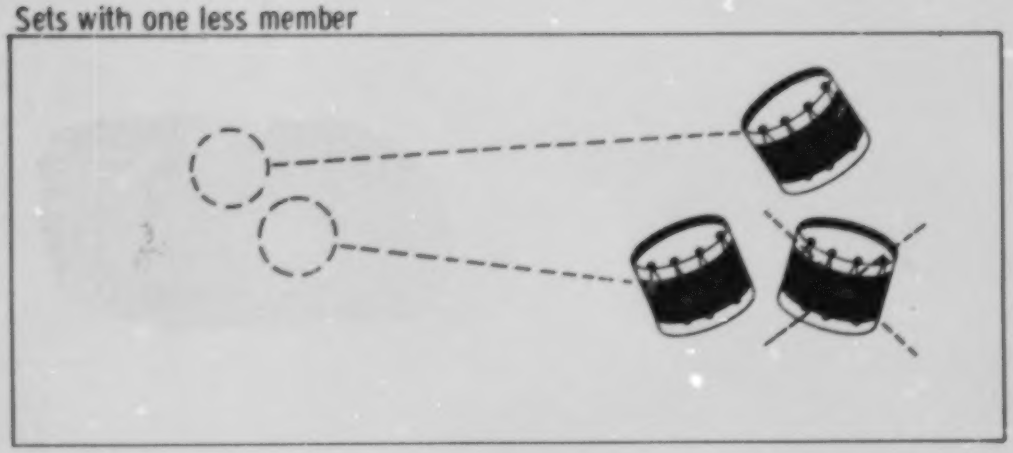
Pages 52 and 53 are teaching pages. Make sure that each child has his book open to the proper page. Display the page in front of the class. Direct the children's attention to the top box. Ask the children,

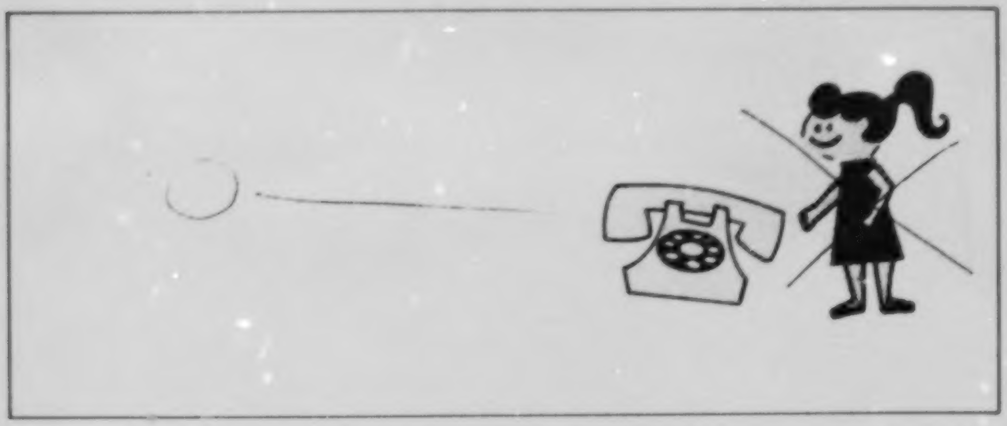
> What is the number of the set of drums? (Three.)
> In the space to the left (point to the space), let us draw a set of rings which has one less member than the set of drums.

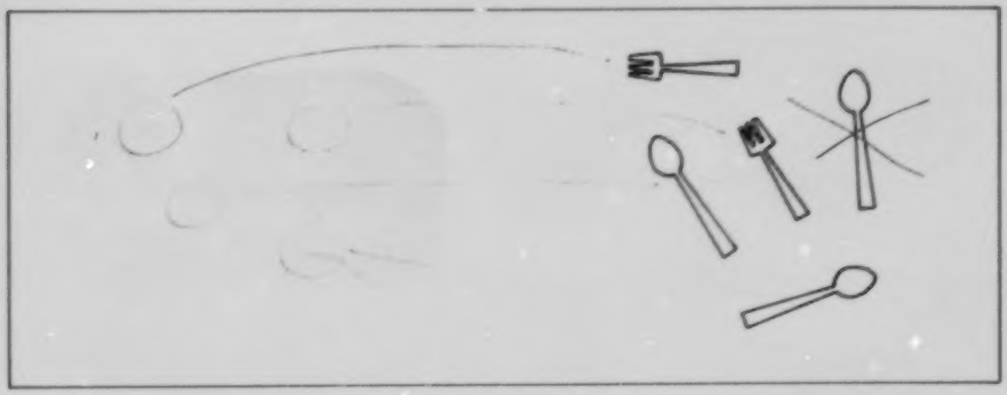
Suggest that the children first mark out one member from the set of drums (tracing over the dotted marks), then draw a set which is equivalent to the remaining members. Proceed in the same manner through both pages.

Pages 54 and 55 are for independent work.

Sets with one less member

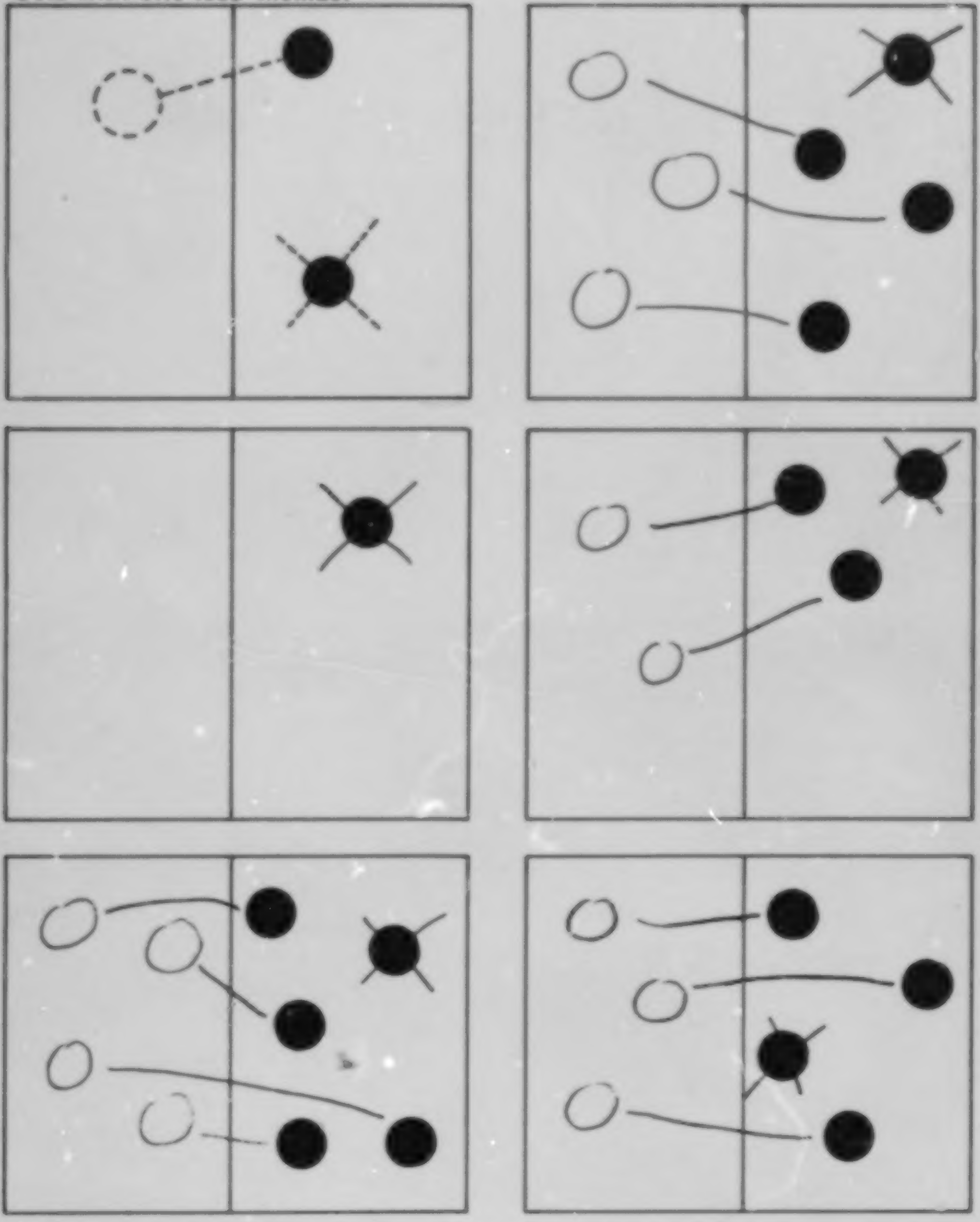




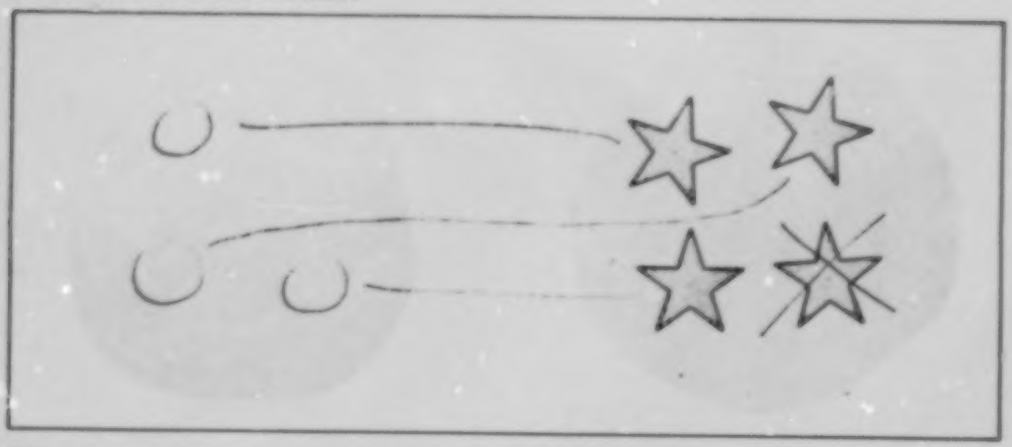


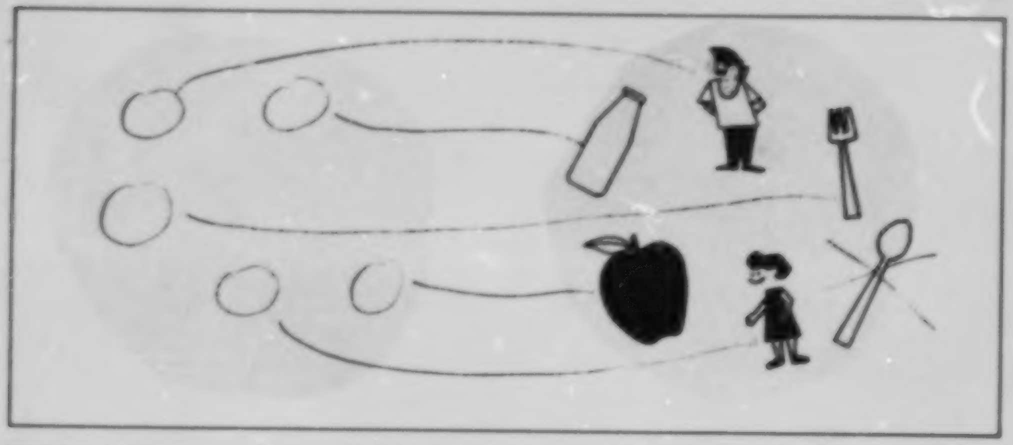
Teaching Page

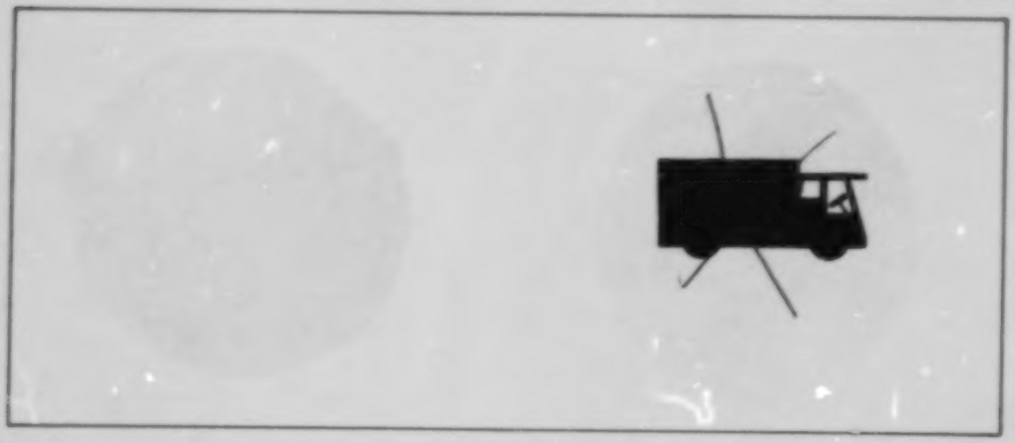
Sets with one less member



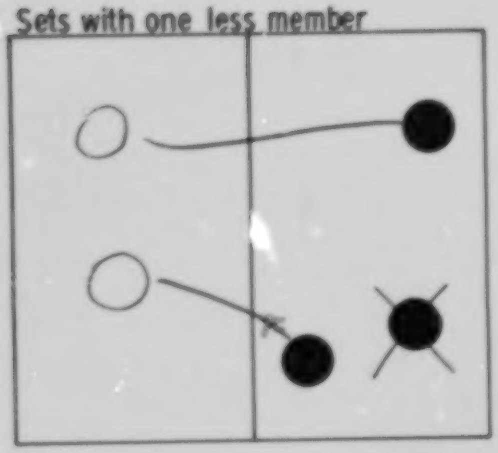
Sets with one less member

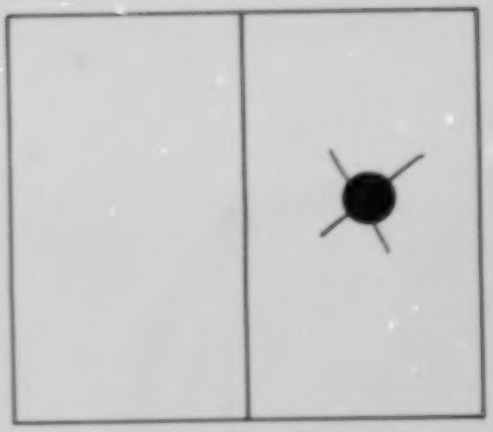


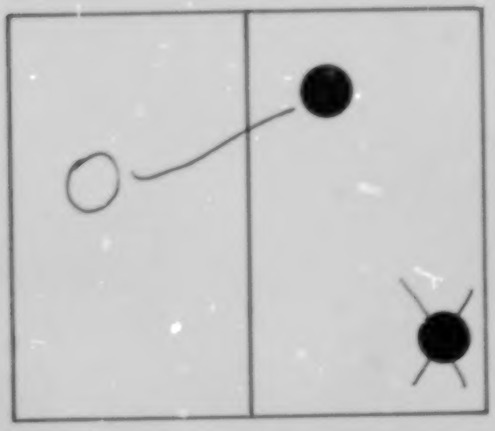


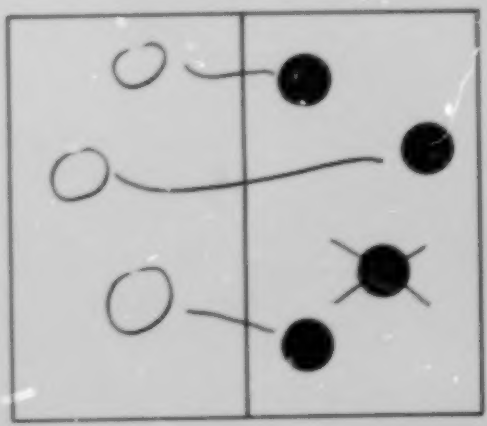


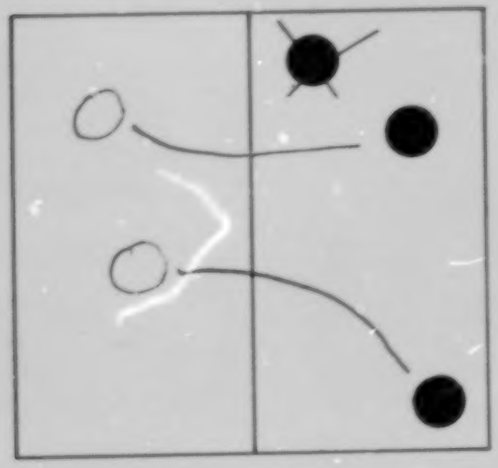
Sets with one less member

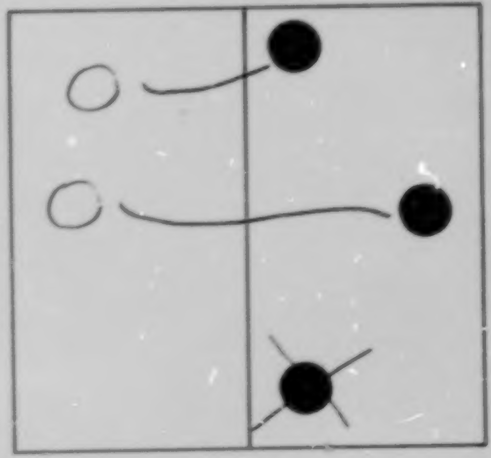












USING THE PUPIL'S BOOK, pages 56-57

Page 56 is a teaching page. Display your page in front of the children and point to the set of two dots in the top box. Ask the children,

What is the number of the set of dots? (Two.)
Is there a set which has one less member than a set of two? (Yes.)
What is the number of that set? (One.)

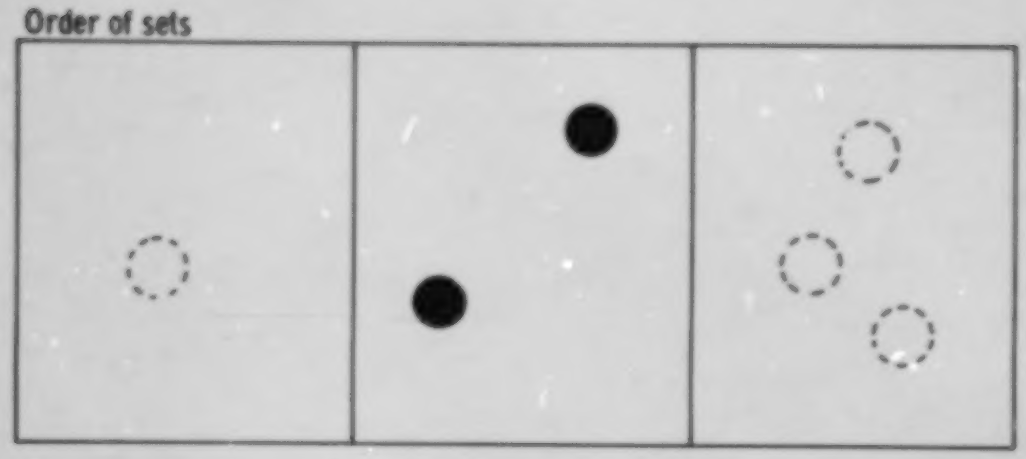
Ask the children to draw a set with one ring in the frame to the left of the given set of two. Now, ask the children if there is a set which has one more member than the given set of two. (Yes.)

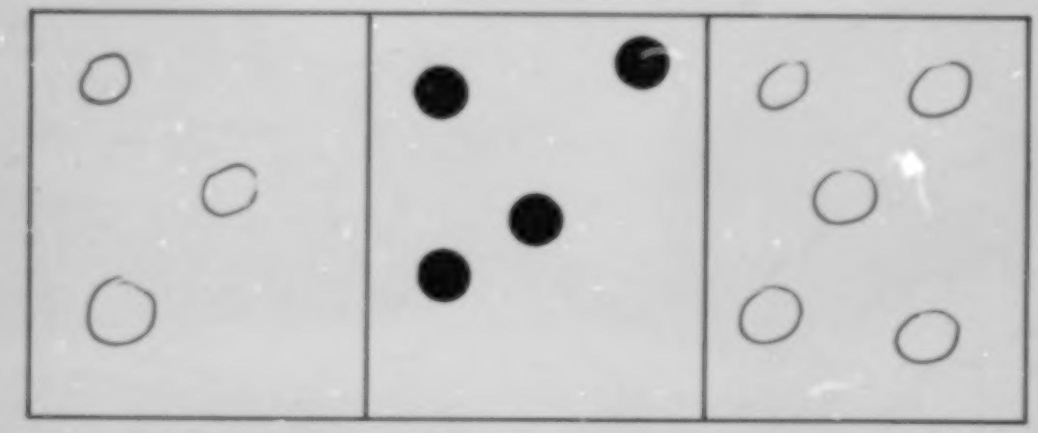
What is the number of that set? (Three.)

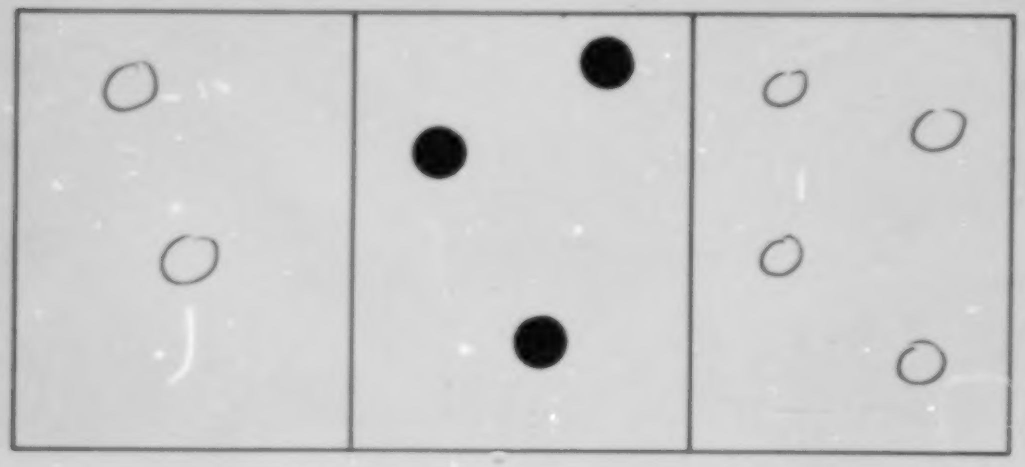
Ask the students to draw a set of 3 rings (or X's) in the frame to the right of the given set of two. Follow the same procedure for the rest of Page 56, then the children can work Page 57 on their own.

Teaching Page

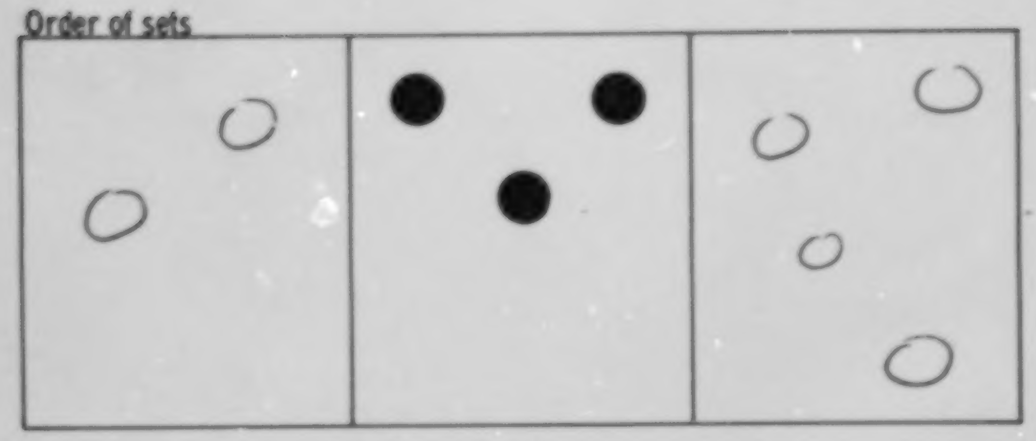
Order of sets

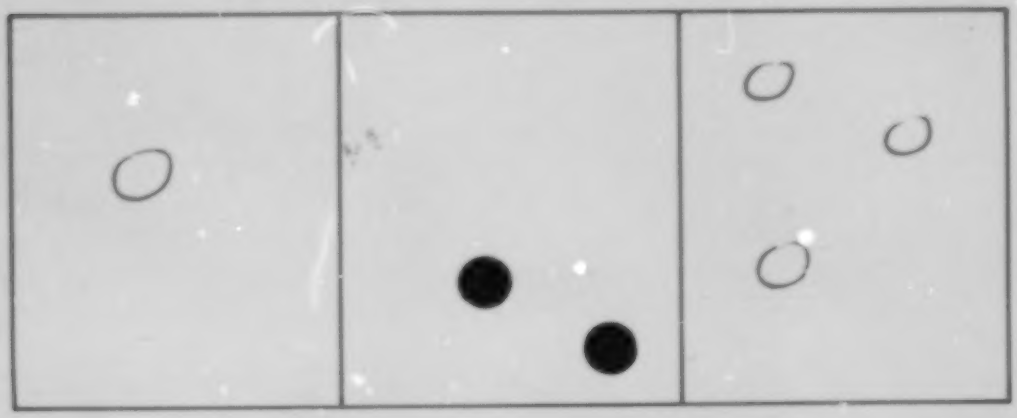


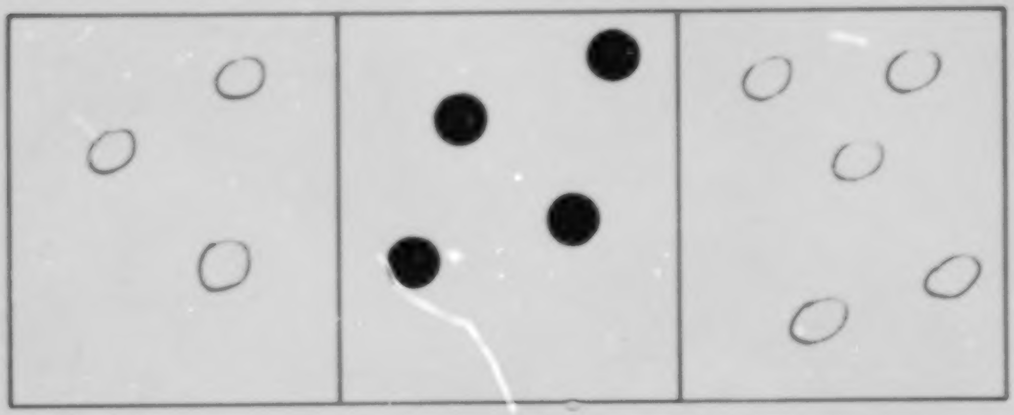




Order of sets

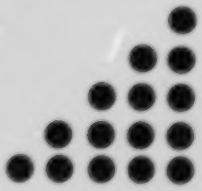






FURTHER ACTIVITY:

Give each child fifteen blocks or other set objects. Ask him to make a set of one, and then make, to the right of this set, a set of three. Ask if there is a set missing between the set of one and the set of three. Continue working until each child has arranged the sets of objects as follows:



Now extend the ordering of set to include those with 6, 7, 8, 9 and 10 members. You may want to take several days to do this. Use a procedure similar to the one you followed in ordering sets with not more than 5 members. Order sets of not more than 6 members, identifying a set of 6 as having 1 more member than a set of five. Then order sets of not more than 7 members, identifying a set of 7 as having one more member than a set of 6. Continue in this way to introduce sets of 8, 9 and 10 members.

If your class cannot order the names of the numbers and count to find the number of members of a set, you will want to provide experiences which will develop this ability. Remember that in beginning to count sets of objects a child must touch each member of a set in sequence, saying in sequence the names of the numbers. (The child is actually pairing the members of the set with members of a set of numbers.) The name spoken when the last member of a set is touched designates the number of members in the set. Later comes the "point and say" stage, next the stage of pointing and thinking and finally, the "look and think" stage.

During counting practice, the child should be led to realize that the arrangement of the set being counted does not matter. That is, if he counts the set, then rearranges it and counts in another way, the same number is obtained. Placing a number of cut-outs on the flannel board and having the children count together, then rearranging the cut-outs and counting again should bring this idea to a level of awareness and verbalization.

The relationship between number and equivalence should be constantly emphasized: If two sets are equivalent then they have the same number of members, and if two sets have the same number of members then they are equivalent. Thus, if a group of boys is counted and the same number of girls is counted, we are sure that we can form couples with no one left over. If we have couples and we count the boys, we need not count the girls since the set of girls and the set of boys are equivalent. Every time these two equivalent sets are displayed and the members of one set are counted, the question of whether the other set should also be counted should be asked and discussed.

The entire development of counting is accomplished by a great deal of practice which should not be confined to the period set aside for the mathematics lesson. Children should count sets of chairs, sets of children, sets of toys, sets of disks, and so on as part of their everyday classroom activities.

Partial counting, that is, recognizing the number of members in a part (subset) of a set and then counting from that number is difficult for youngsters even though they know the number names in order. Once the order of the number names through ten is established you may begin the development of the concept of partial counting. For example, in ordering sets you may check the number of members of a set of six by counting. Then comment that there is another way. Since the set before 6 has 5 members it is easier to think: one more than 5 is 6. Similar comments about a set of 7 might be: 7 is 1 more than 6 and 7 is 2 more than 5.

This type of procedure should be followed consistently even though a change in the counting practices of the children may not be observed for a long period of time.

USING THE PUPIL'S BOOK, pages 58-59

These pages are concerned with the recognition of the cardinality of sets of 6 - 10 members. Two pages are devoted to each number. The same procedure may be used with each page.

Page 58 is a teaching page. Display your page in front of the class. Read the title to the class (pointing to the title as you read). Now, point to the set (top left) of apples and drums. Ask the children,

Can we find the number of this set of apples and drums? (Yes.)
How can we find the number? (By counting.)

Suggest that the class count out loud. Tell the class to mark out each member as it is counted. Point as you count, starting with the drums first.

What is the number of the set? (Five.)
Suppose we counted the apples first, would we get the same number? (Yes.)

Whenever possible, emphasize the point that the order in which a set is counted makes no difference. Since the set does not have six members, it should be marked out with a large X. Do the same thing for each set on the page. When the children are through, all sets except those sets with 6 members should be marked out.

Page 59 is for independent work.

With a little help in the form of instruction, the children should be able to complete Pages 60 - 67. If they have difficulty then introduce each number as you did on page

Pages 68 - 69:

Procedure is the same as for pupil's Pages 48- 51.

Pages 70 - 71:

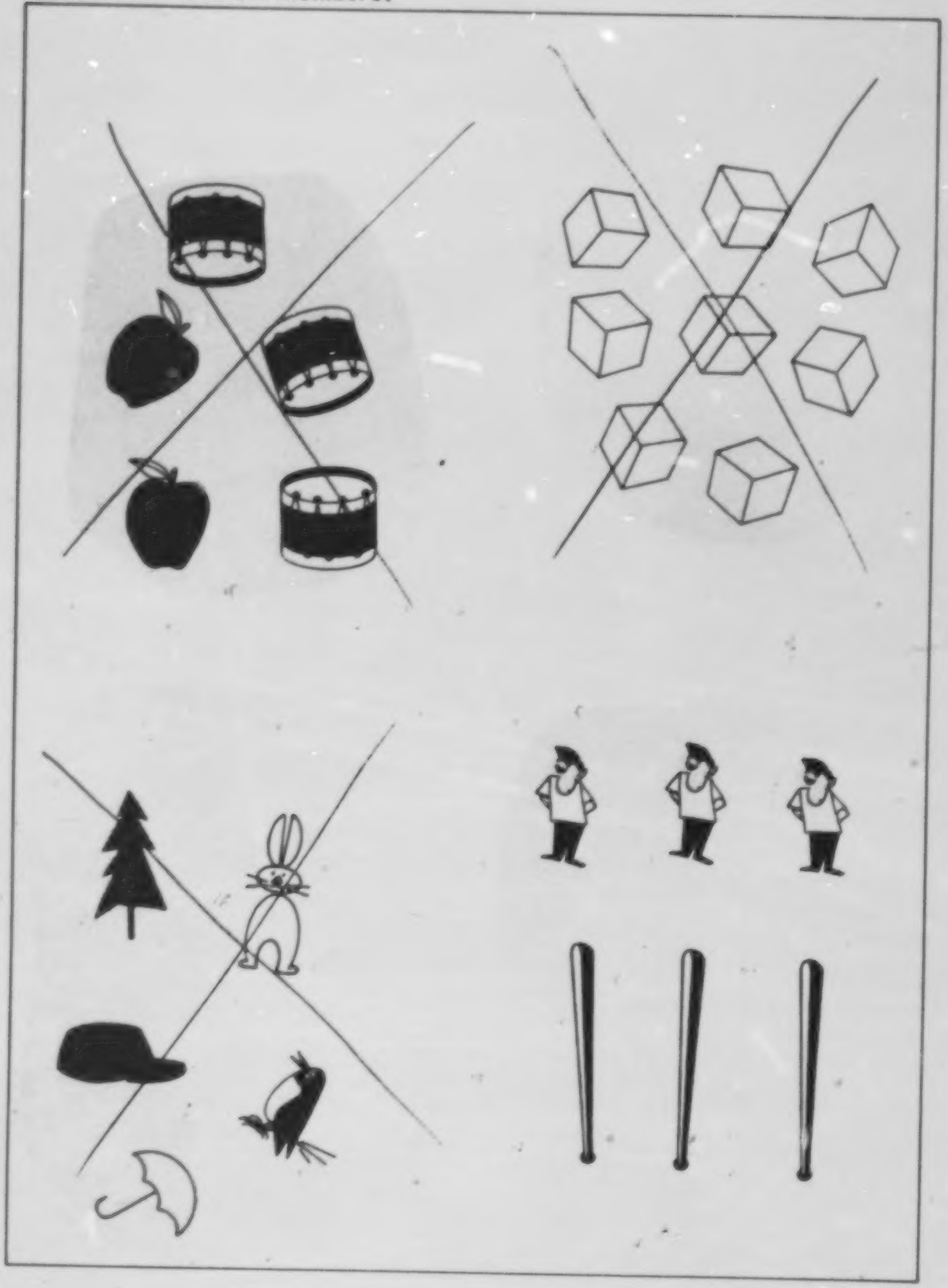
Procedure is the same as for pupil's Pages 52 - 55.

Page 72:

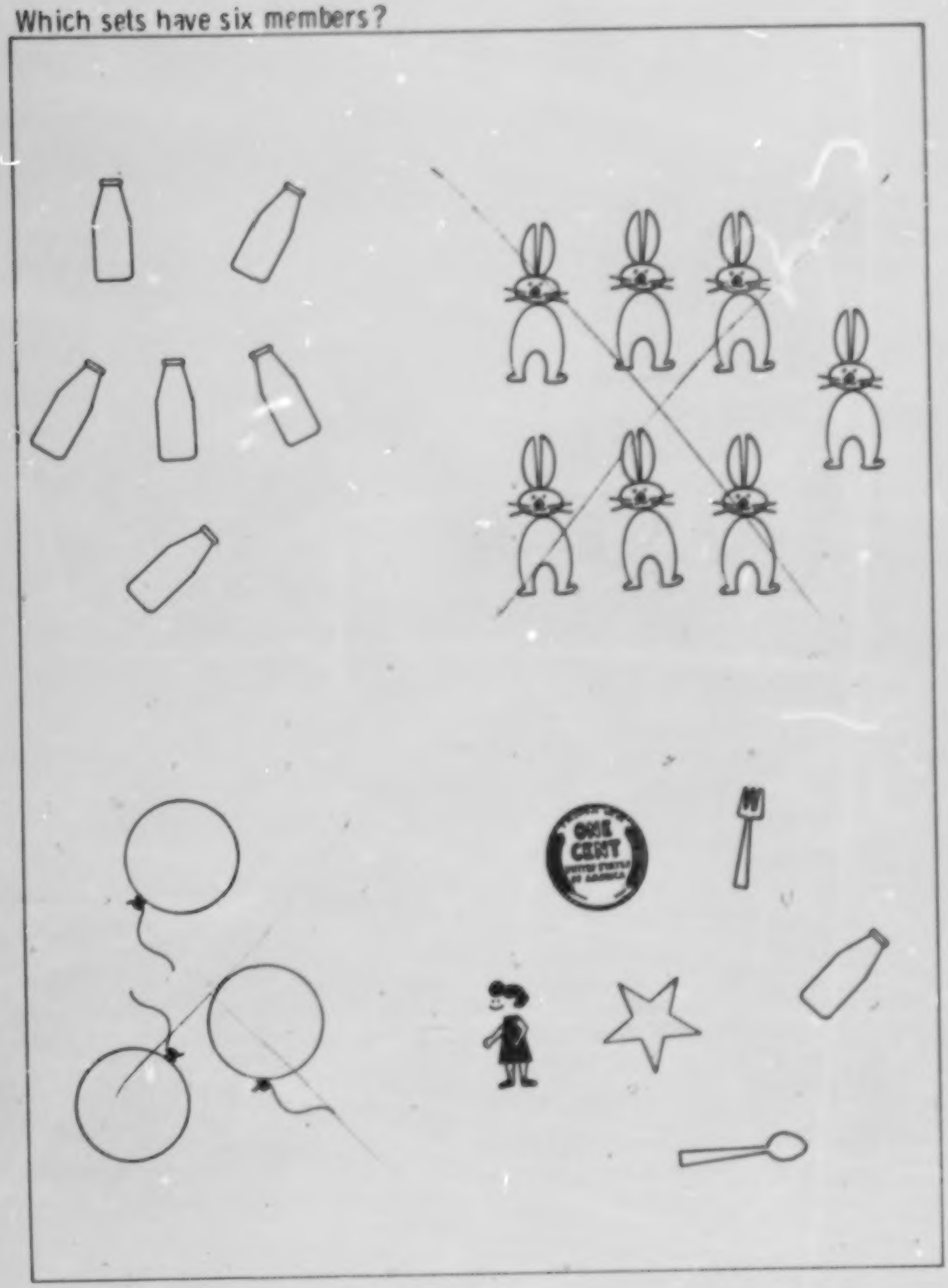
Sets should be drawn so that the three sets are in order from left to right.

Teaching Page

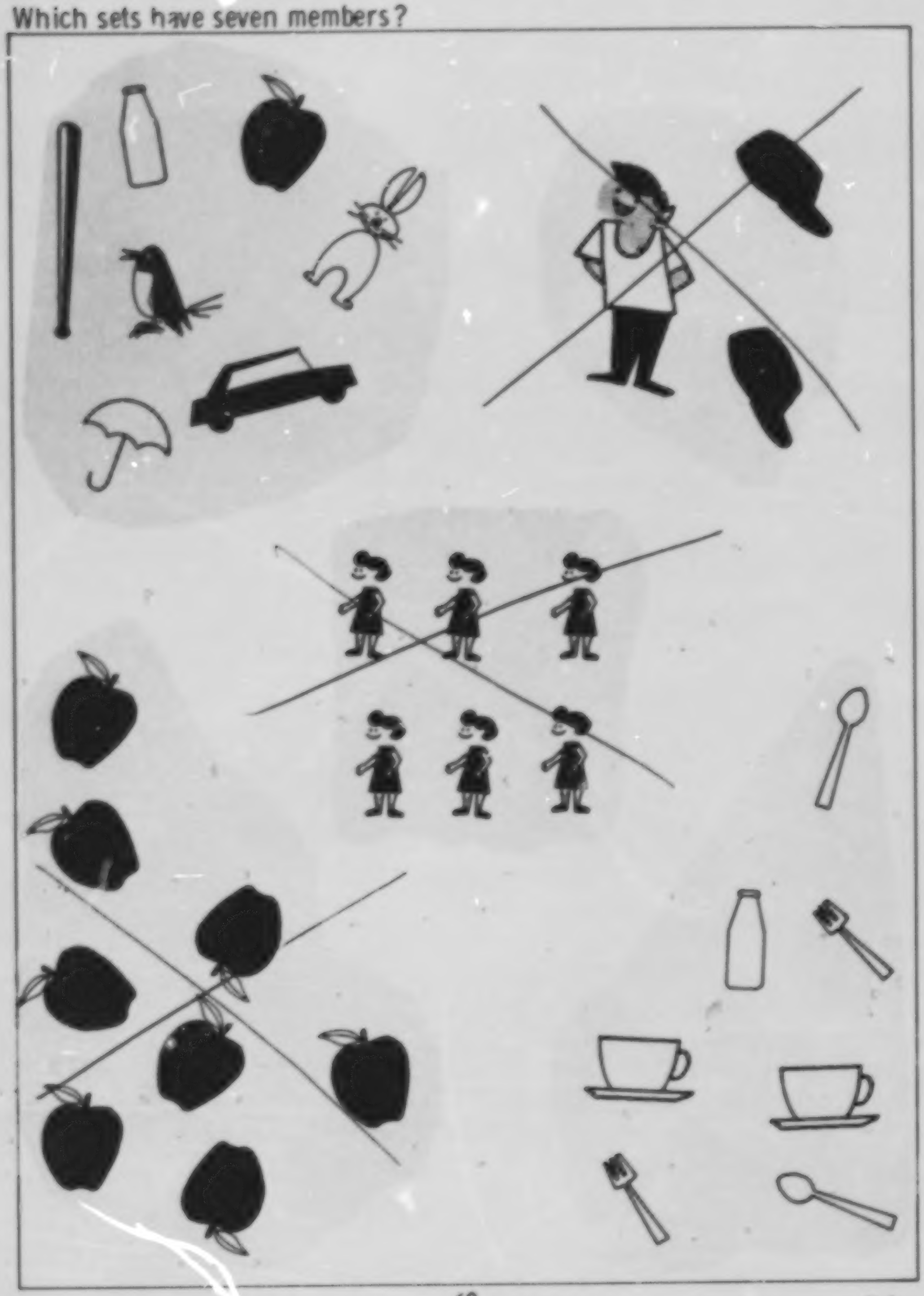
Which sets have six members?



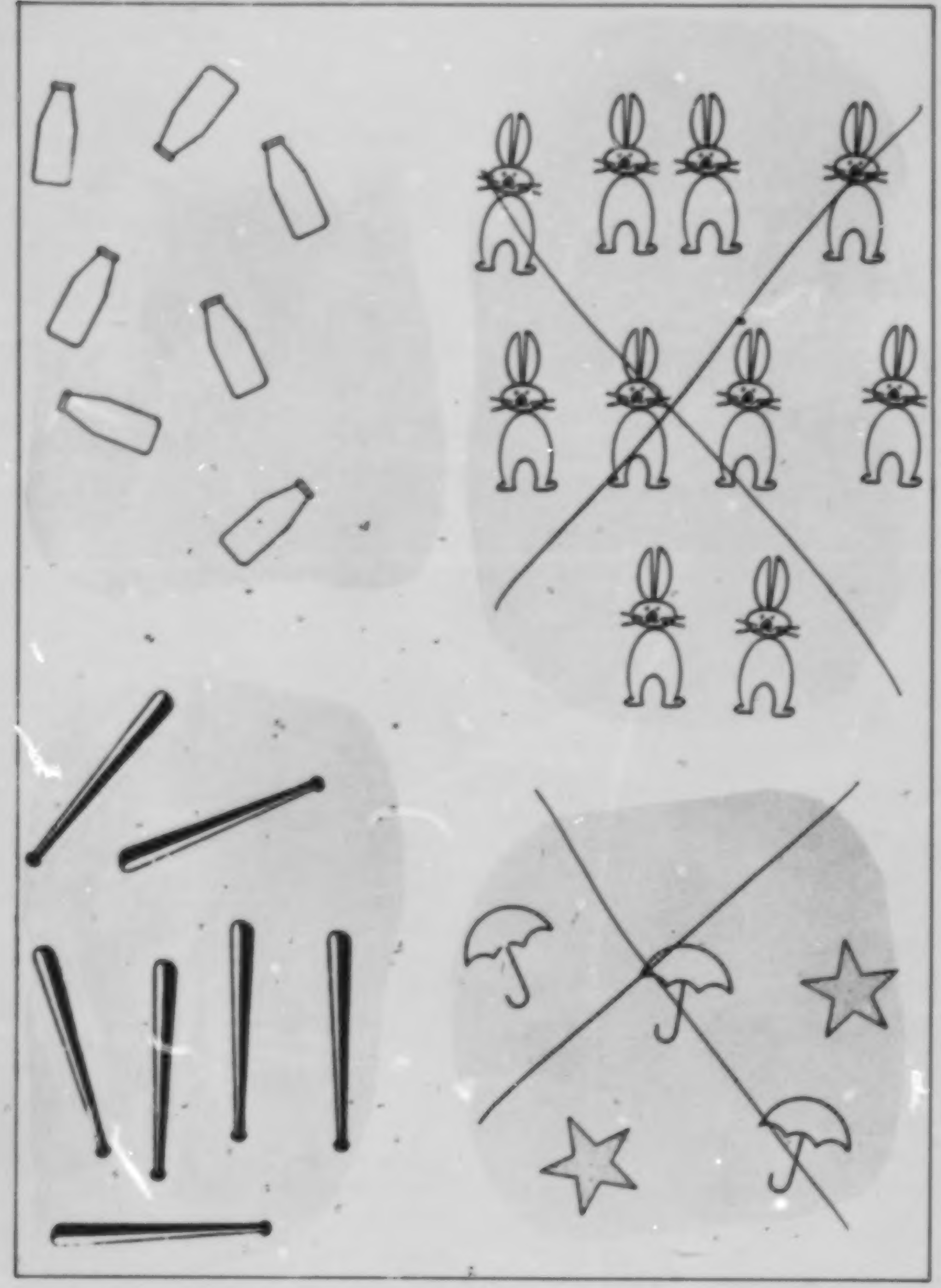
Which sets have six members?



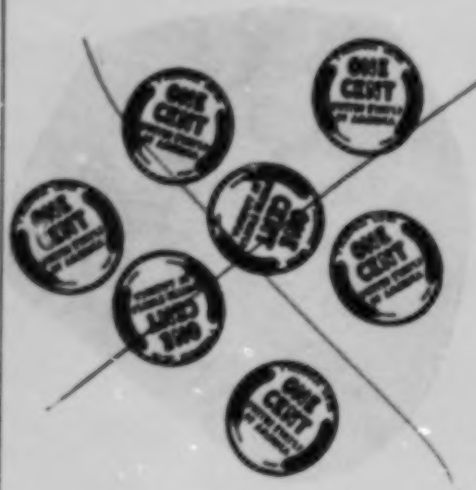
Which sets have seven members?

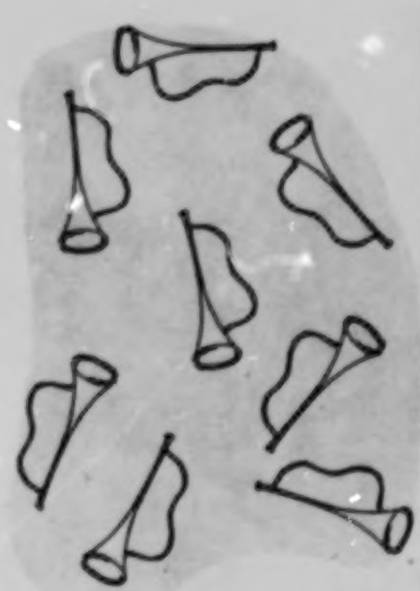


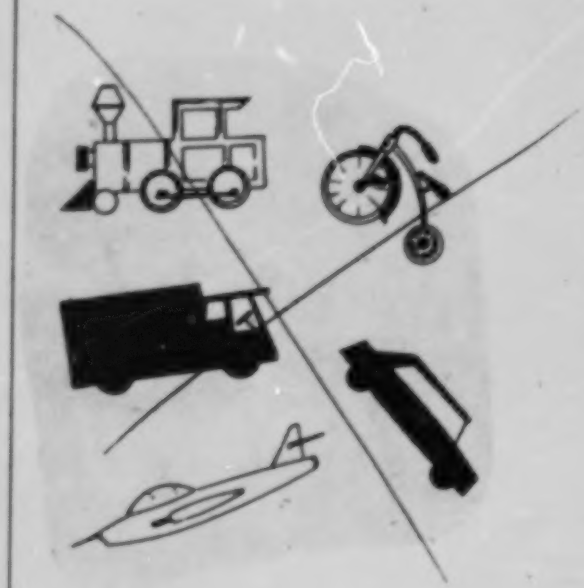
Which sets have seven members?

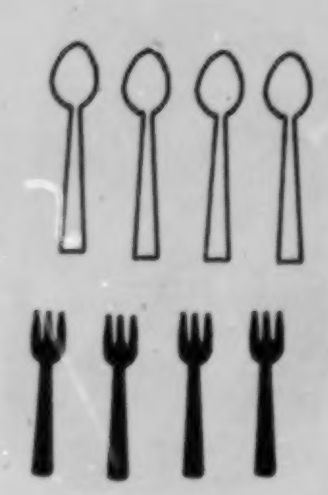


Which sets have eight members?

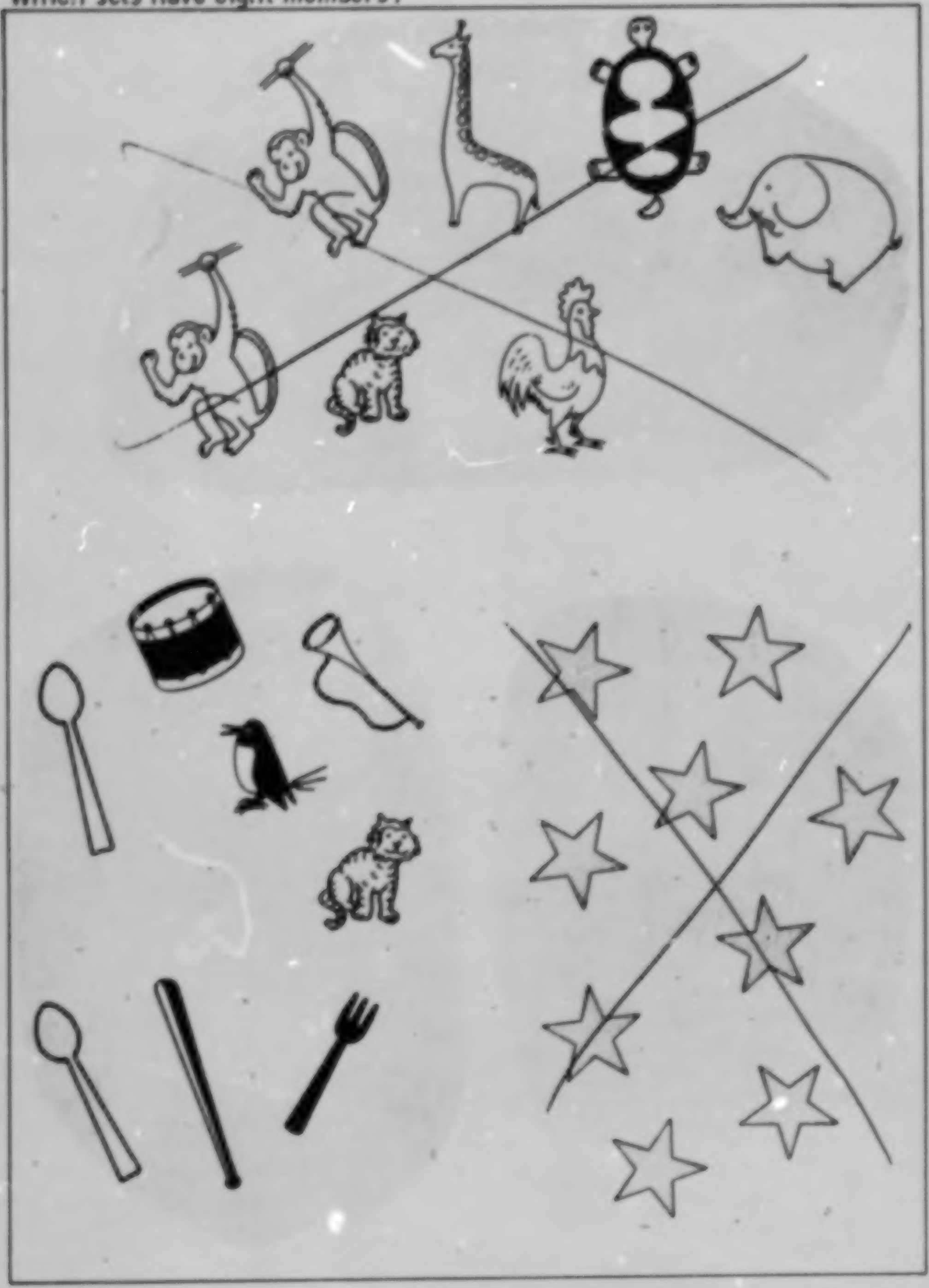




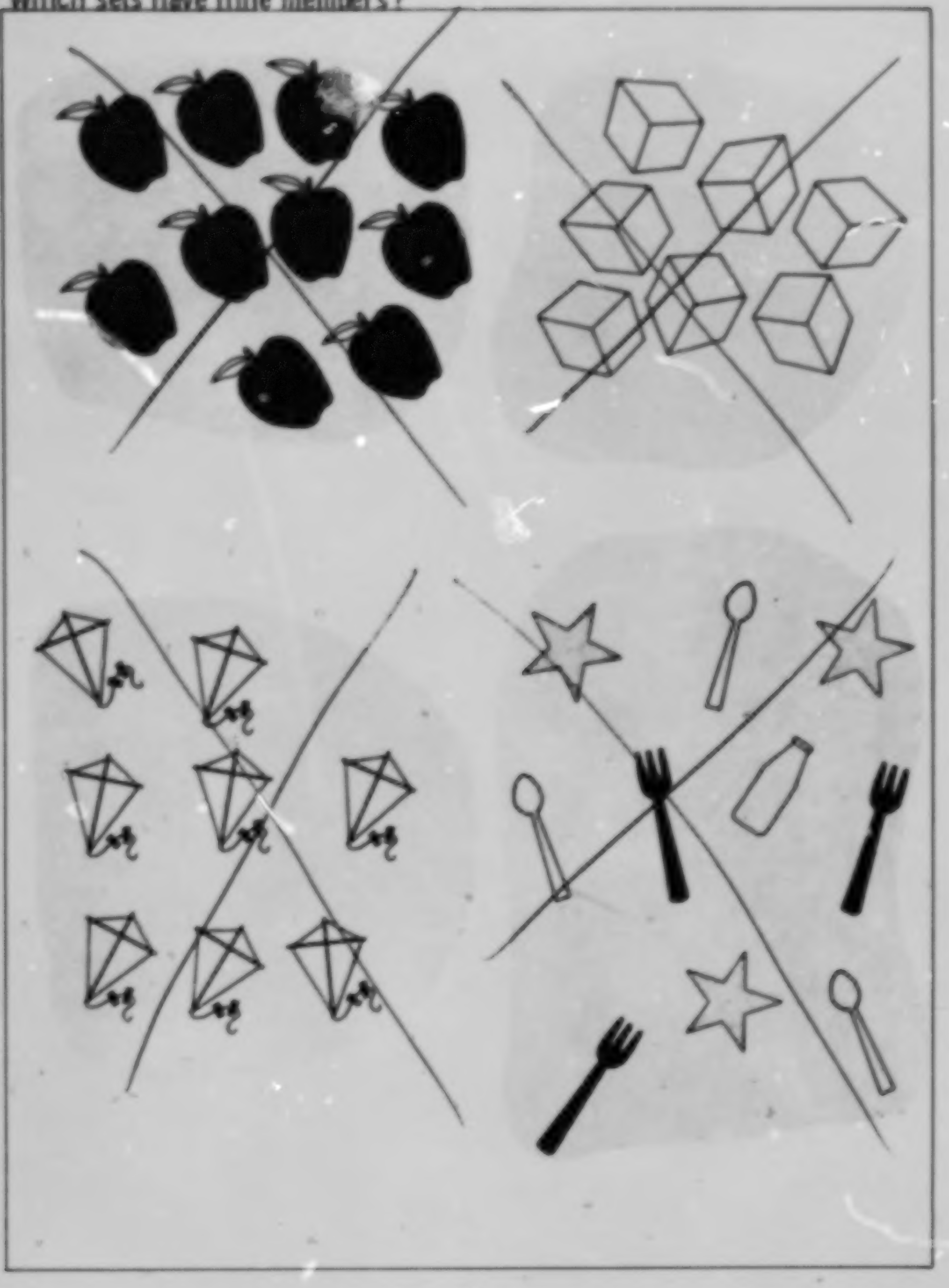




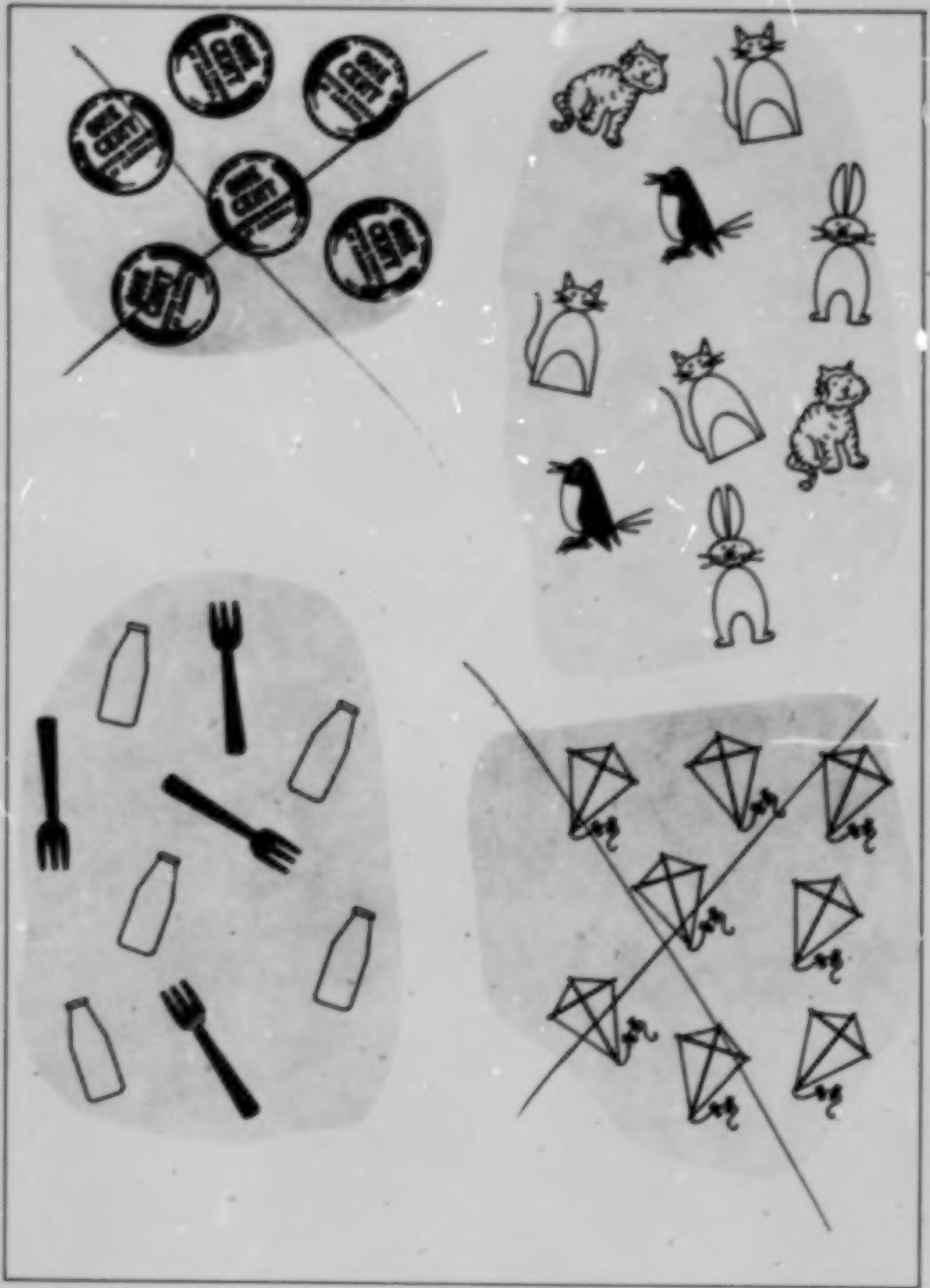
Which sets have eight members?



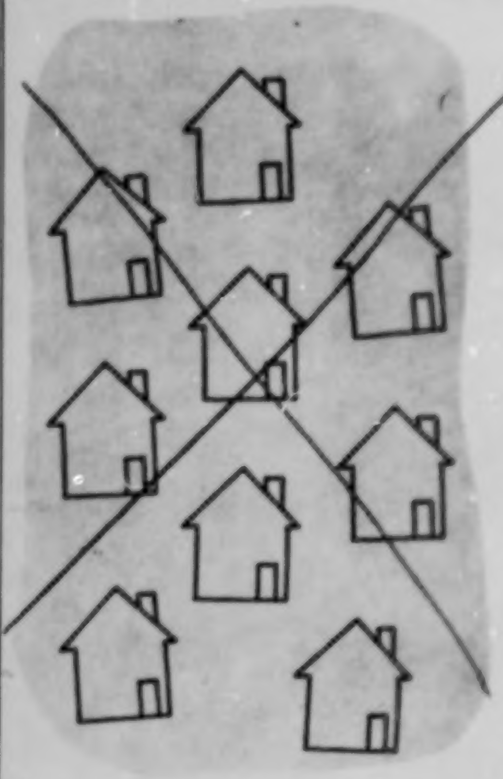
Which sets have nine members?

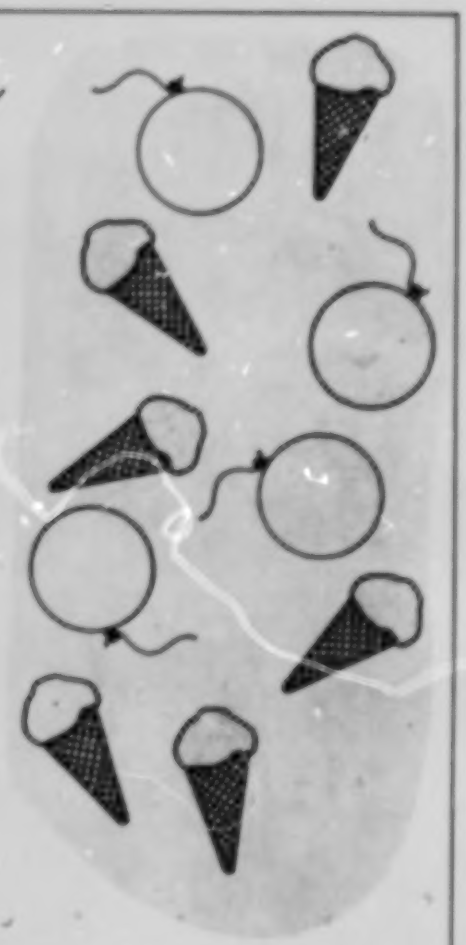


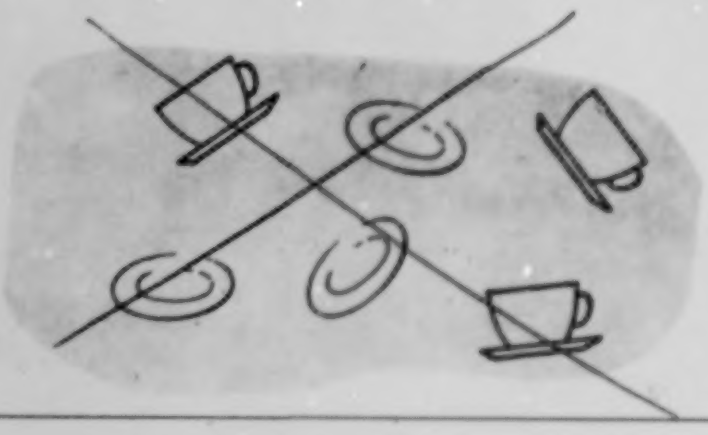
Which sets have nine members?



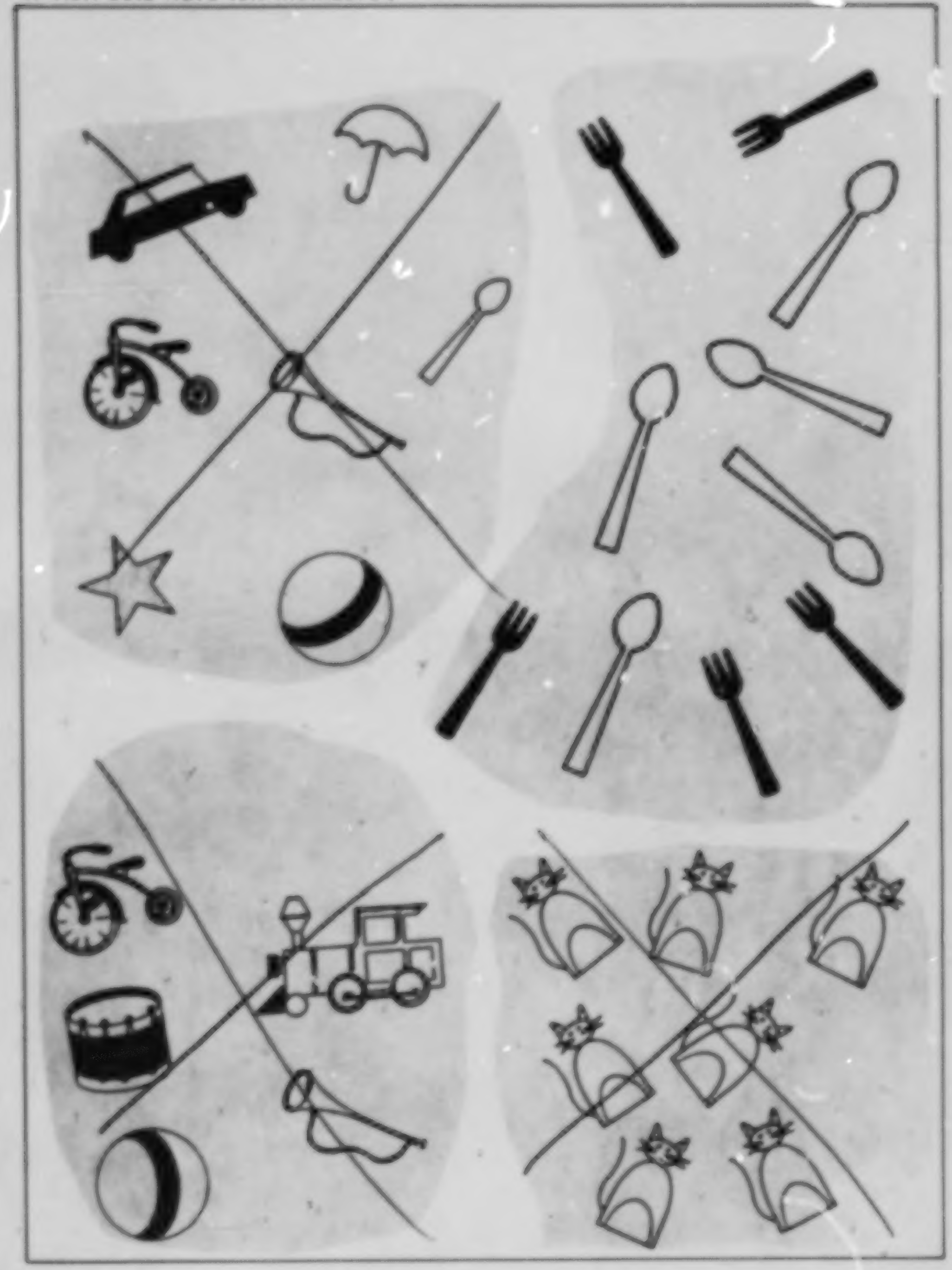
Which sets have ten members?



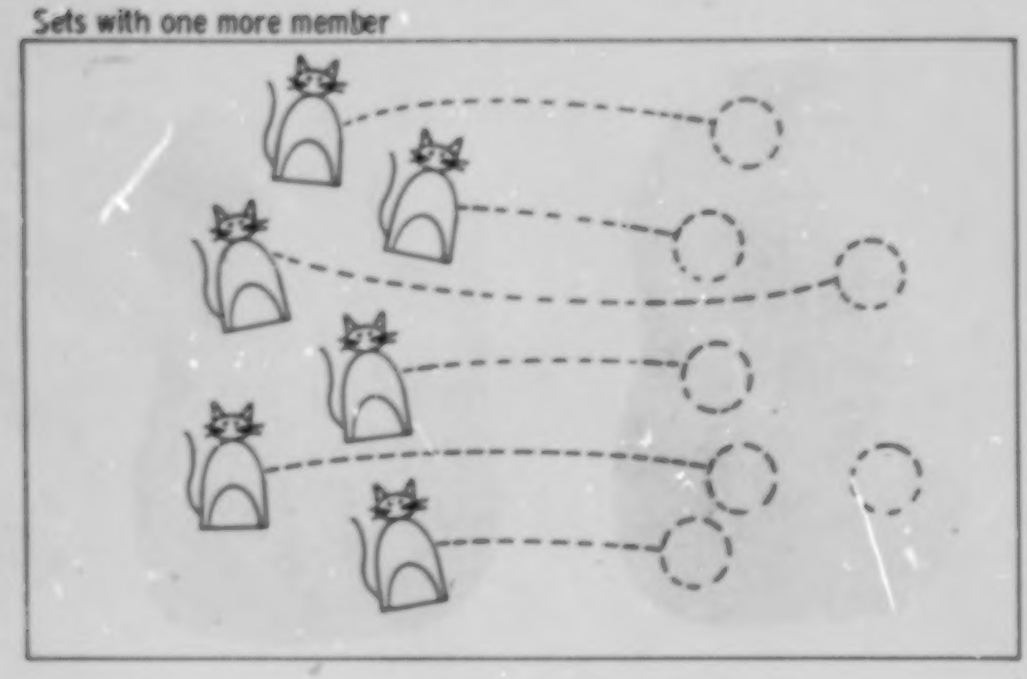


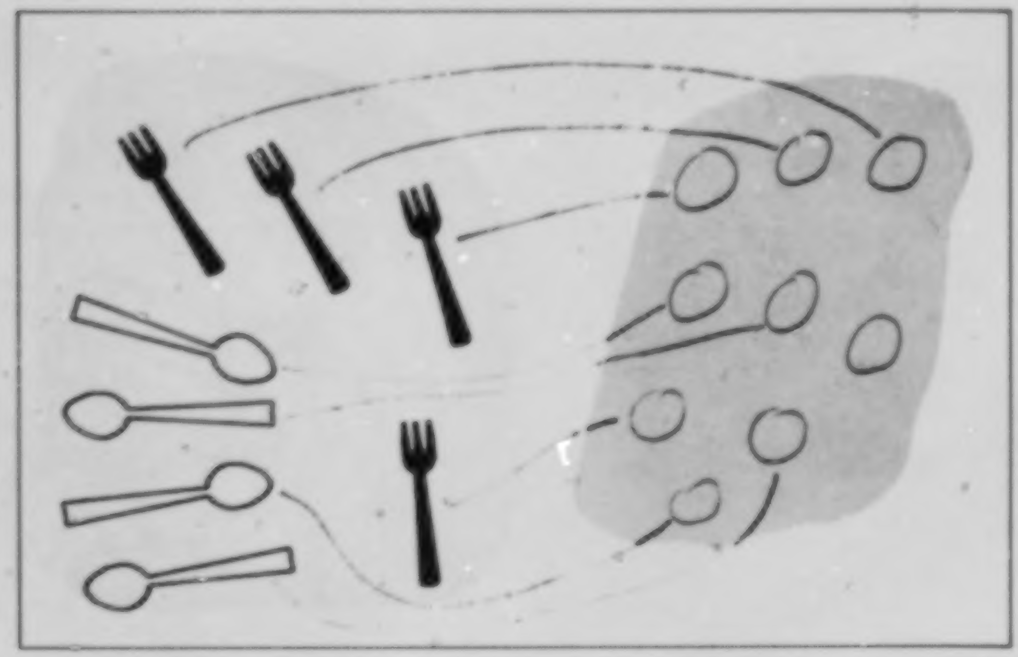


Which sets have ten members?



Sets with one more member





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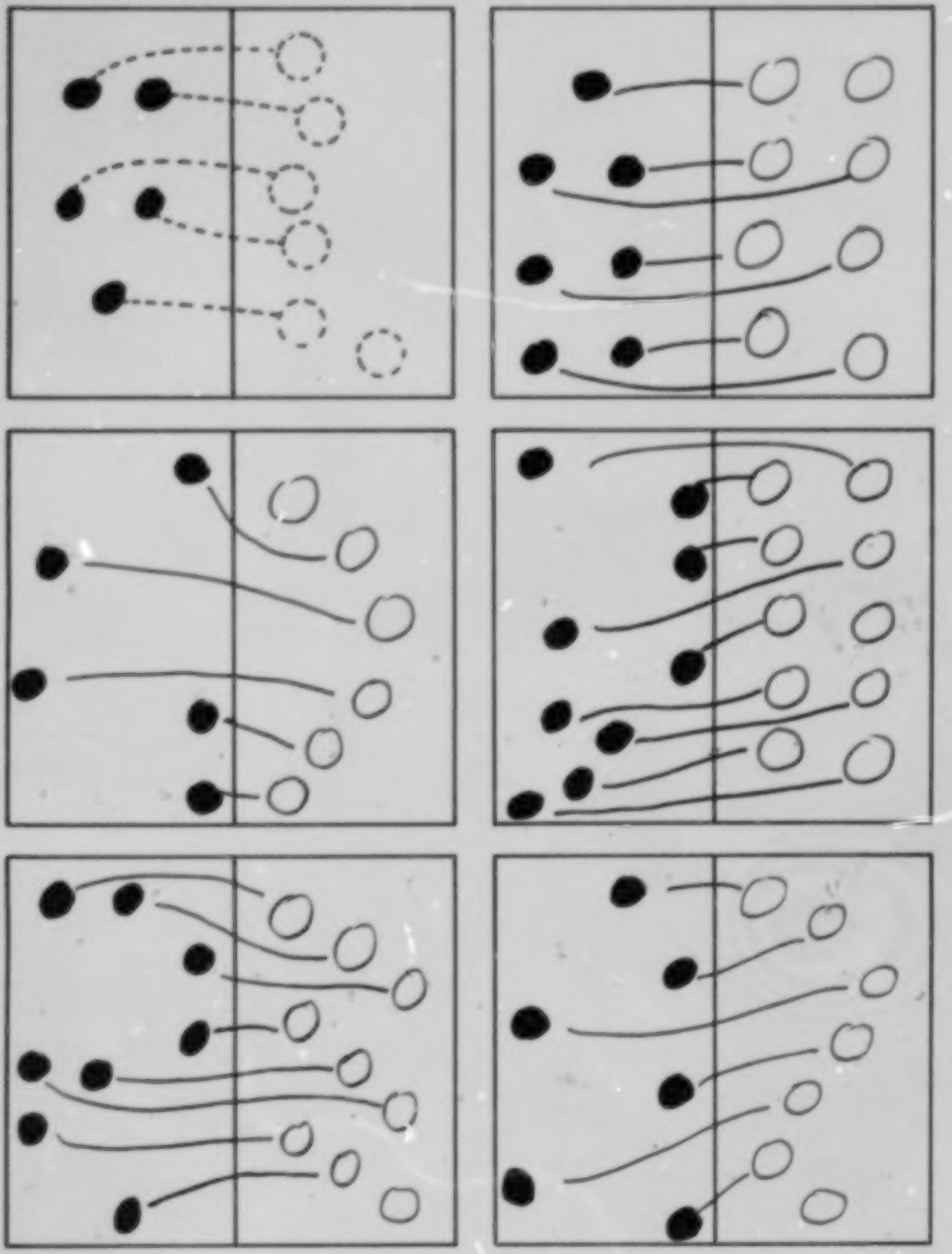

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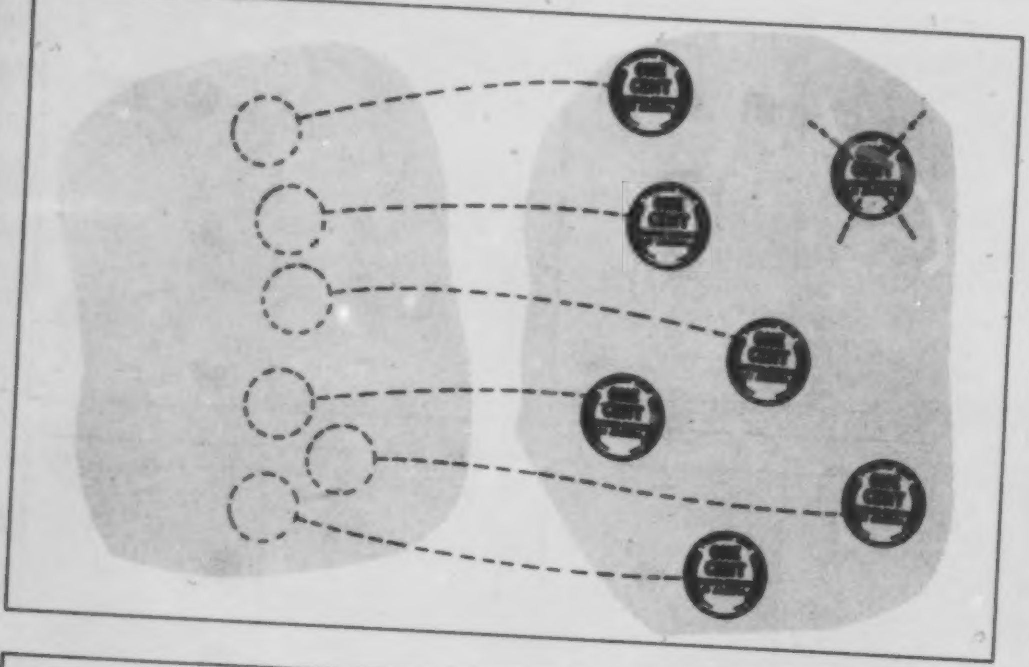
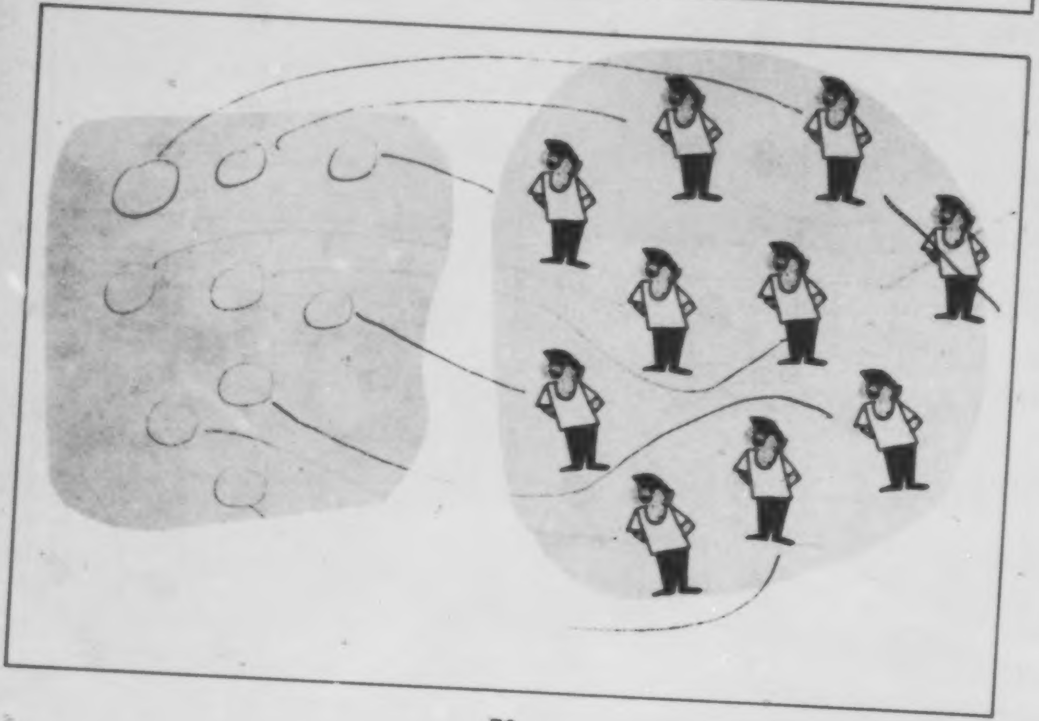
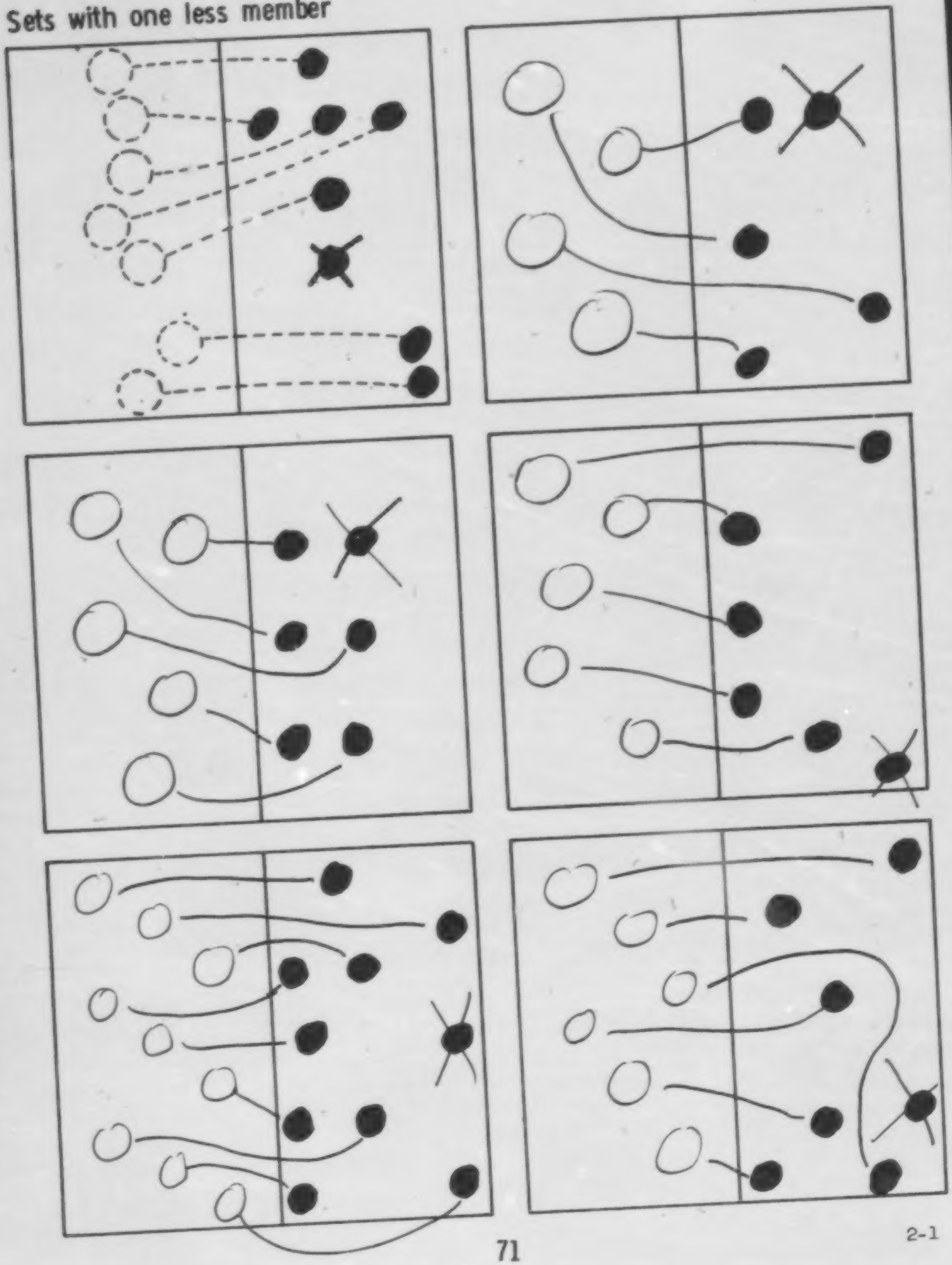
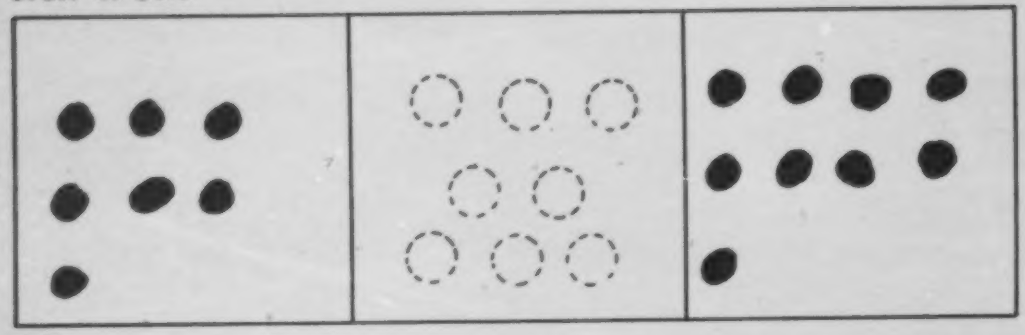


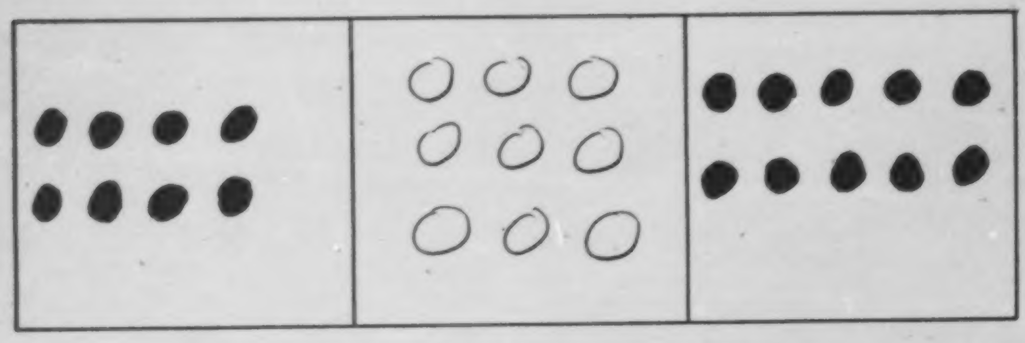


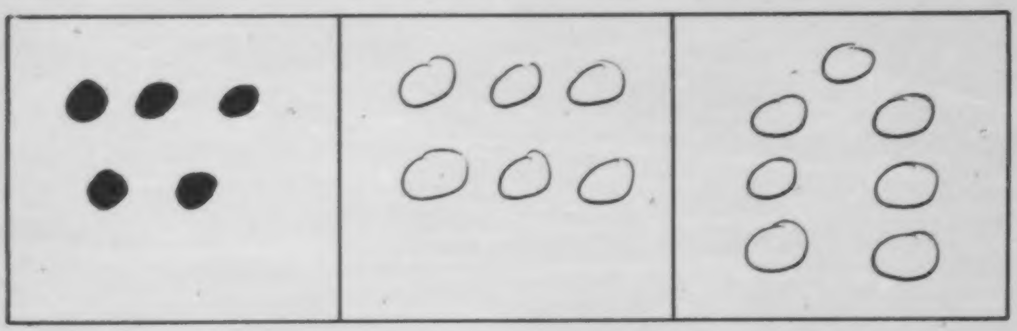
Sets with one less member

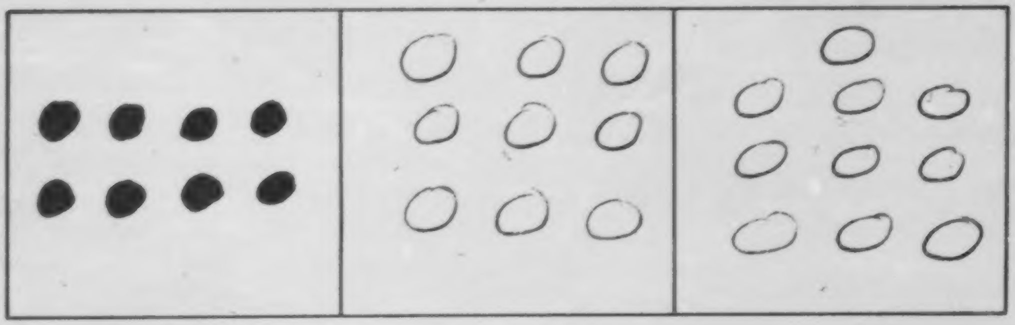


Order of Sets





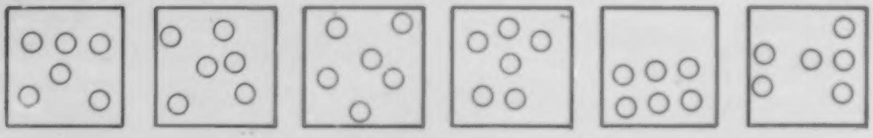




FURTHER ACTIVITIES

As a further activity, scatter a set of objects with 6, 7, 8, 9, or 10 members on a small rug or towel. Ask a child to pick up a part (subset) of the set which he can name immediately and begin his counting from that number. Each child should be encouraged to pick up the subset he identifies first rather than to look for a particular subset.

The teacher can make cards that show sets of 6, 7, 8, 9, and 10 in a variety of arrangements. Each set should have a minimum of six arrangements so that children do not just learn a particular pattern. Sets should be constructed on the same color of card, preferably with the objects drawn alike to prevent association of a given set with a given object shape or card color.



Display a set of these cards on the chalkboard ledge. Children may be encouraged to order the sets during an activity period. A variety of games can be developed that will help children learn to recognize the number of these sets. For example, two teams may be selected. Hold up the cards one at a time and ask the children in turn to identify the number of the set. Encourage the children to "think out loud" when they are finding out how many.

Although most children will count each member of a set of 7 members, some other possible responses are: Five, one more is six, and one more is seven; Two, three, four, five, six, seven; Five and two more is seven.

2-2. THE NUMERALS 0 THROUGH 9

OBJECTIVE: To associate the written numerals 0 through 9 with their corresponding numbers, and to order these numerals.

VOCABULARY: Numeral.

MATERIALS: Peg board, flannel board, numeral cards (0 - 9), blocks, books, counter, brushes.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Part I - The numerals 0 through 5

Tell the children that many kinds of marks can be used to tell how many. Make a set of marks (/ /) and tell a child to bring a set of books with that many members to the front of the room. Ask another child to bring a set of blocks whose number of members is the same as this set of marks (:+: :+:). Ask another child to get X X brushes. Identify the number in each set as it is brought to you. (2.)

Ask each child to place on his desk a set of objects which has as many members as the one displayed. Introduce a numeral card for 2. Explain that the figure is called a numeral, that it is a special mark for the number two, and that it is read "two".

- Show a word card with Betty (or the story character from your reading series) on it.

Is this word really Betty?
Does the word have curly yellow hair and a smiling face? (No.)
It is just Betty's NAME.
A numeral is the name of a number.
This (pointing at the numeral card) is a name for two.
"Two" (written on the chalkboard) is another name for two.

Place a set of one and a set of two on the flannel board.

Are these sets equivalent? (No.)
Why not? (They don't have the same number. The second set has more.)
Are the sets ordered? (Yes.)
What is the number of the second set? (2.)

Ask a child to select the flannel cut-out of the numeral and place it under the set of two members. Then hold up the flannel numerals for 0 and 1 and ask the children if they know which numeral names 1. After the numeral has been identified, ask a child to place it under the set of one member.

Is there another set represented on the flannel board? (Yes, the empty set.)

What number tells us how many members are in the empty set? (Zero.)

Hold up the flannel numeral for zero and explain that this is the numeral that names the number zero. Ask a child to place the cut-out on the flannel board to the left of 1.

We have ordered these sets and the numerals that name the number of each set.

Look carefully and see if you can tell me some way that might help you remember which numerals name zero, one, and two.

Hopefully, some child will point out that the numeral for zero looks like the letter "O" and that the numeral for one is a straight mark. Consequently the numeral for the number two is left for consideration. It is helpful to make large numerals on the chalkboard. Then as you face the chalkboard, use your arm to trace these symbols in the air and ask the class to imitate you. Such an experience provides readiness for the actual writing of the numerals.

USING THE PUPIL'S BOOK, pages 73-82

Page 73 is a teaching page. Be sure that each child has his book open to Page 73. Read the title of the page (pointing to the words as you read). Direct the children's attention to the numeral 0 in the middle of the page. Remind them that the numeral is the symbol we use for the number zero which is the number of the empty set. Now point to the set of one telephone and ask,

Is the number of this set 0? (No.)

Let us mark out this set with a large X.

Have pupils trace over the dotted marks. Direct students' attention to the empty set below the telephone. Ask them, Is the number of this set zero? (Yes.)

Have the students draw a line from the empty set to the 0 numeral. Proceed in the same manner for each set on Page 73. All sets with elements should be marked out, and lines should be drawn from the empty sets to the zero symbol.

Pages 74 and 75 are for independent work. However, be sure that the pupils recognize the symbols 1 and 2 before doing these pages.

Page 76:

Ask the students to draw a line from each set to the numeral that names the number of the set.

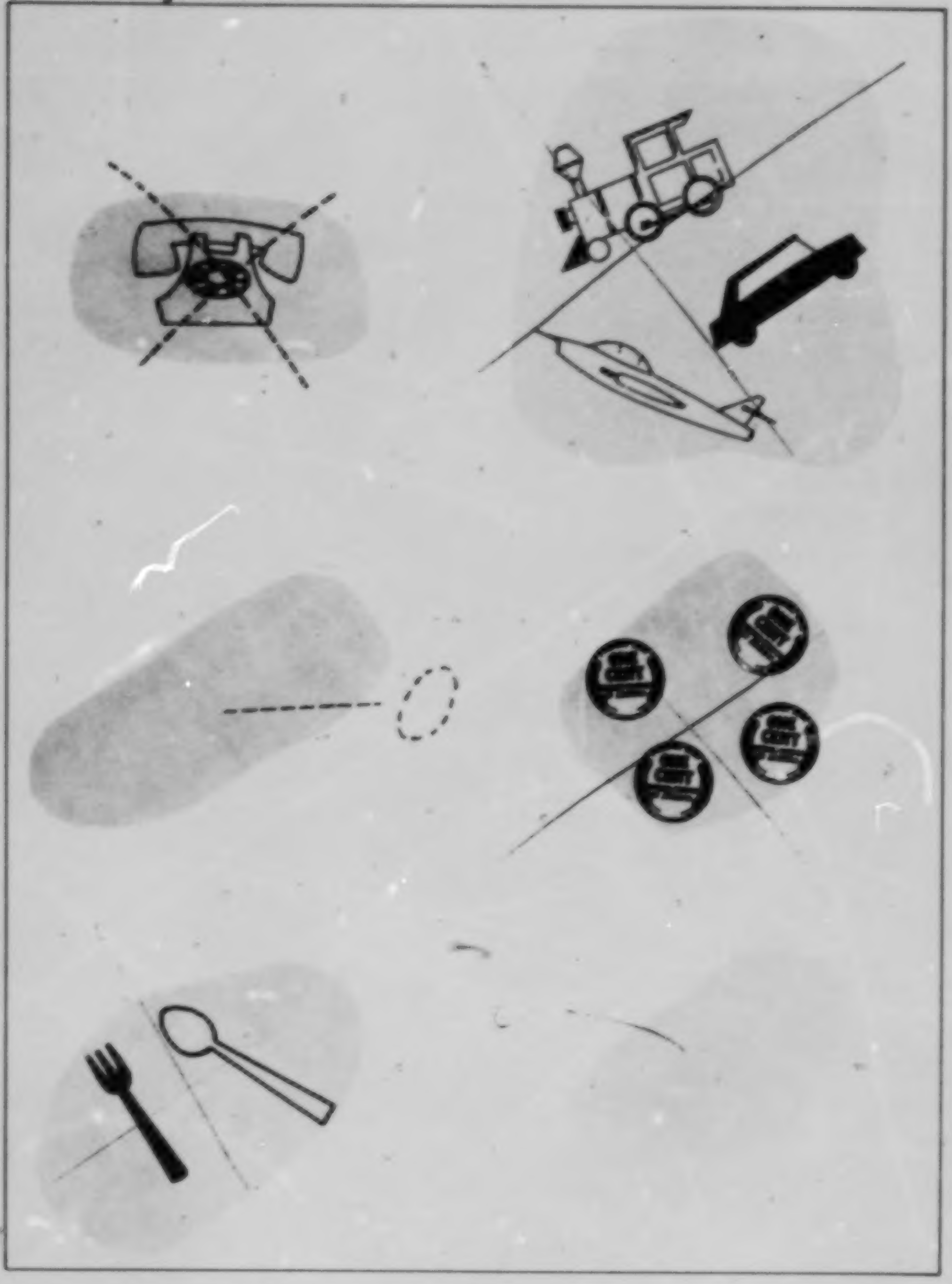
USING THE PUPIL'S BOOK, pages 77 - 82

These pages require the same set of directions as for pages 73 - 76.

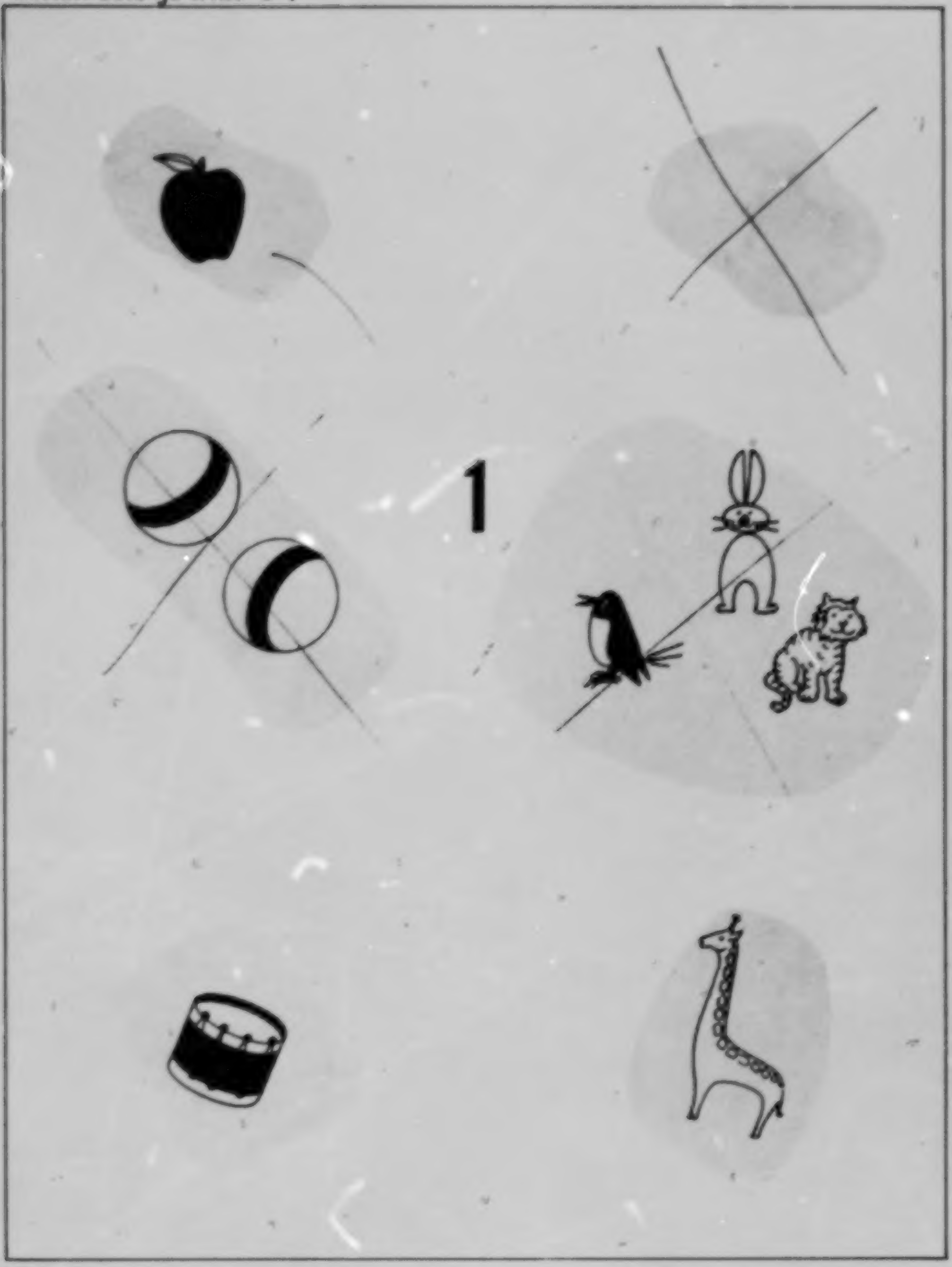
- The numerals 3 - 5 may be introduced one at a time on succeeding days. Order sets previously identified and label with the appropriate numeral. Then construct the next set and introduce the numeral that names the number of the set. Discuss the distinguishing characteristics of the numeral. For example, 2 has a rounded part and a straight part and 4 is made up of straight parts. Rhymes which emphasize the formation of the numeral have been found effective. Stand in the back of the room as the children trace the numerals in the air. If any child has a tendency to make the numeral incorrectly, check this immediately. Emphasize the beginning point in making the numeral.

Teaching Page

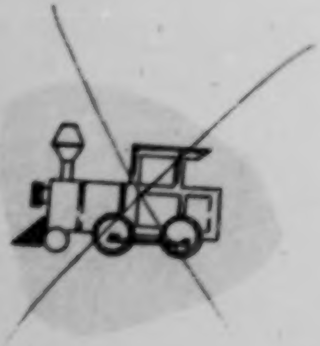
Which sets go with 0 ?

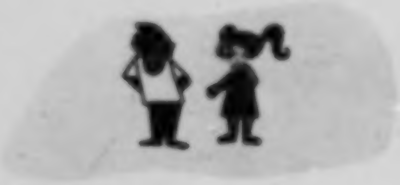


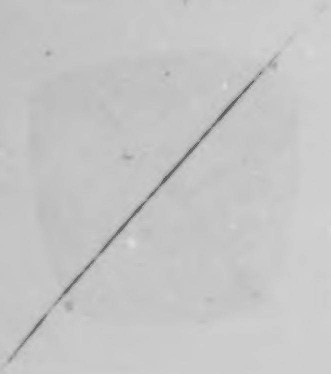
Which sets go with 1 ?



Which sets go with 2 ?







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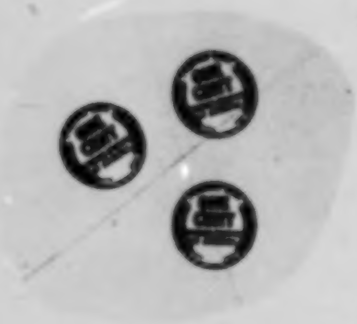
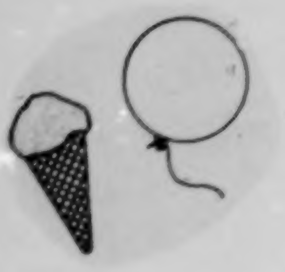
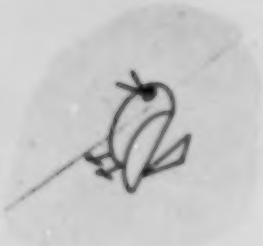
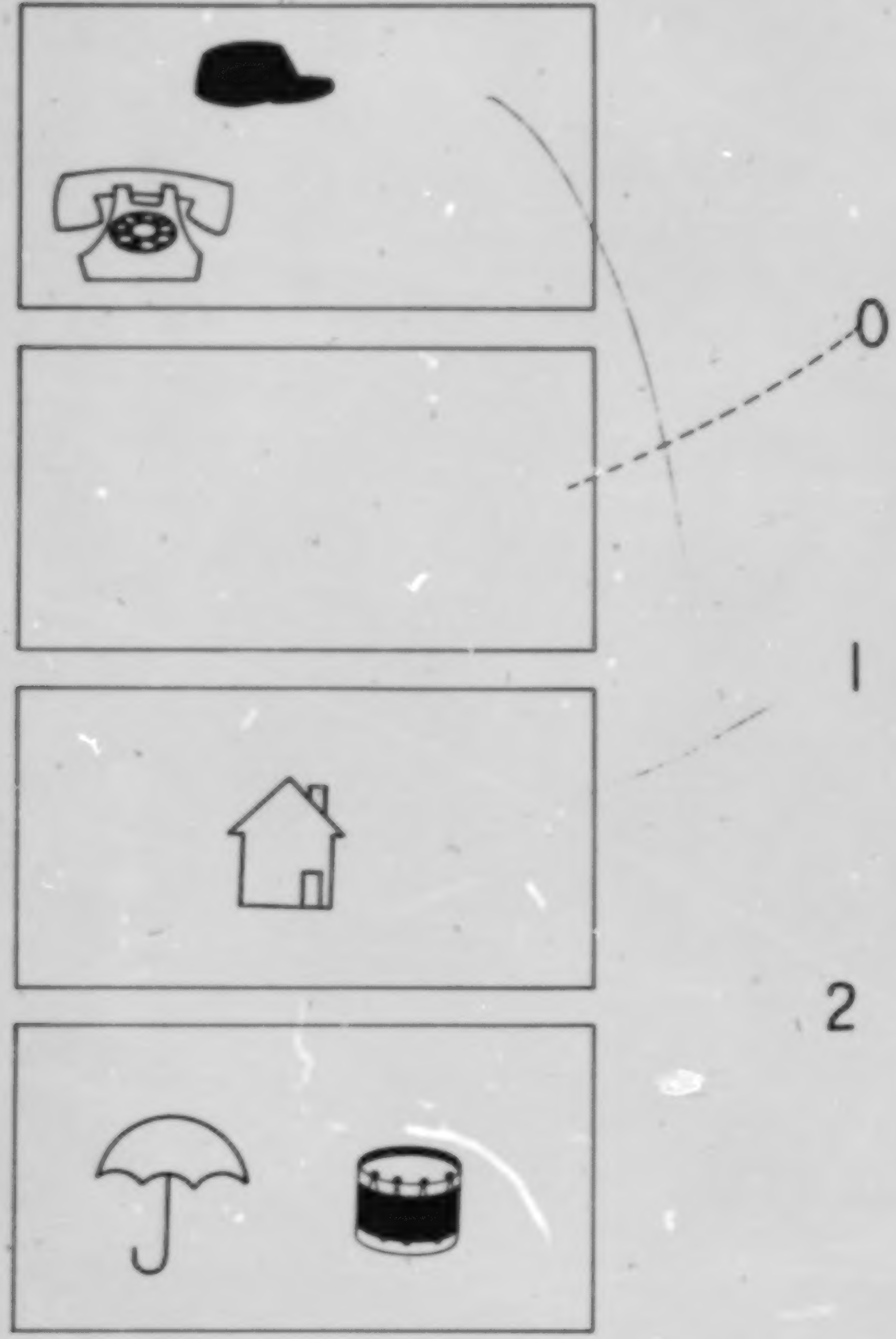






Pair each set with its number name



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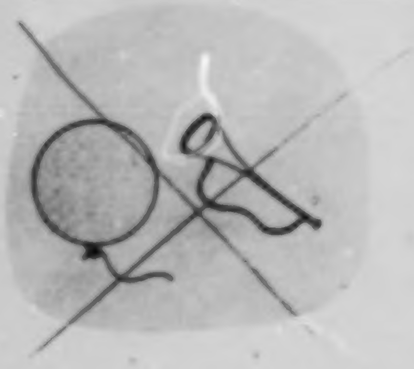

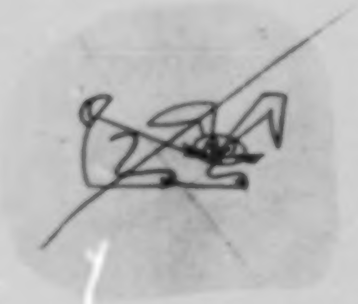

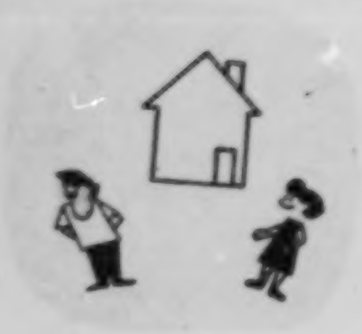

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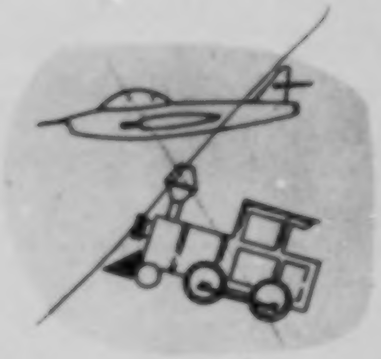
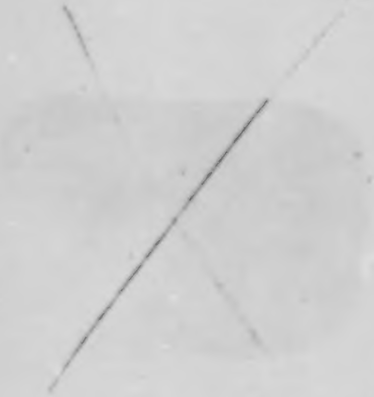
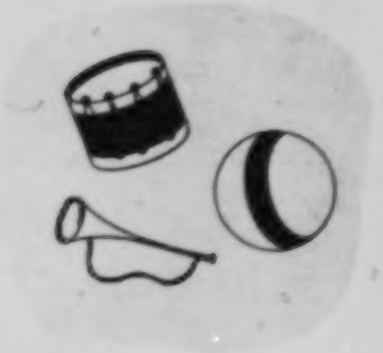
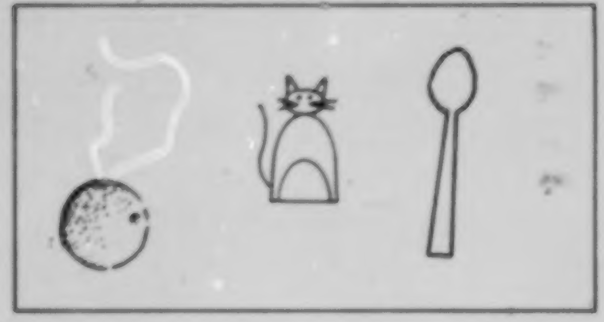






Pair each set with its number name



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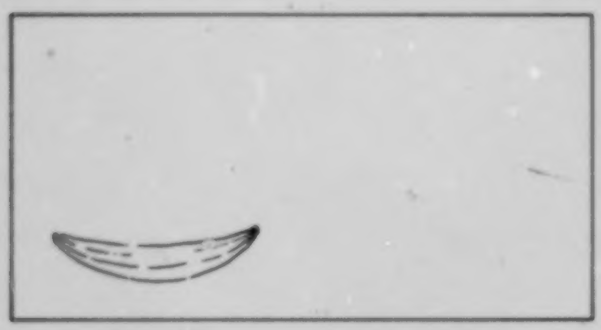

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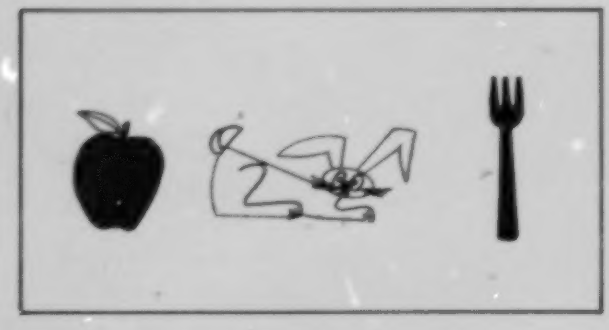

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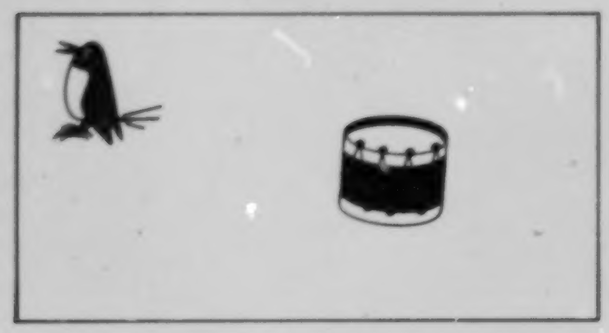

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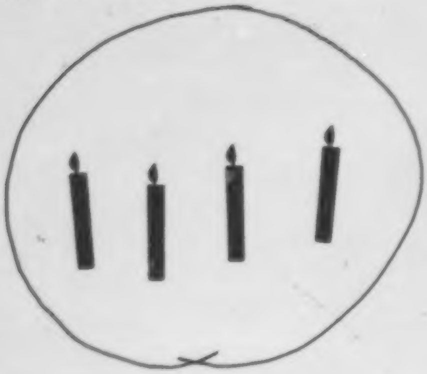
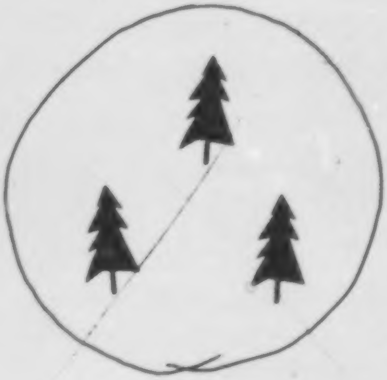
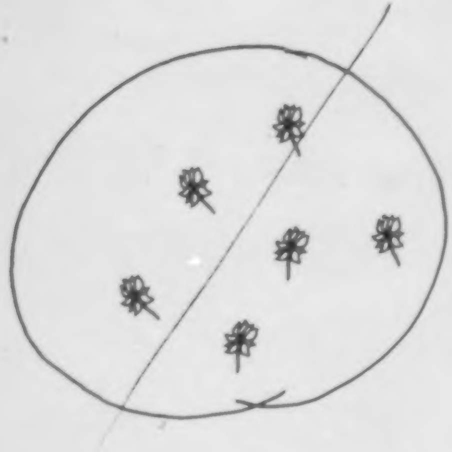
Which sets go with 4?







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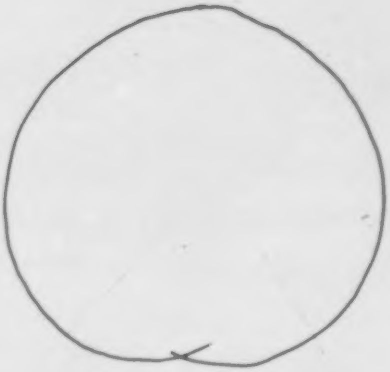
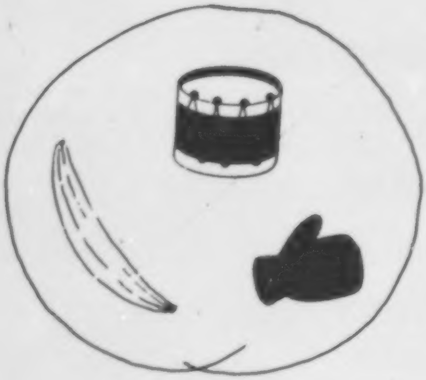
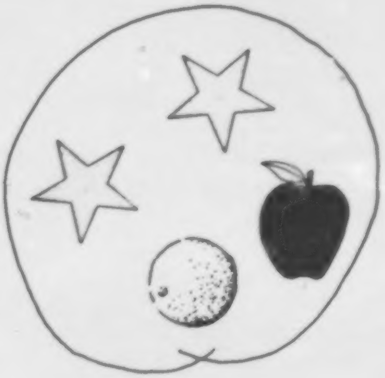
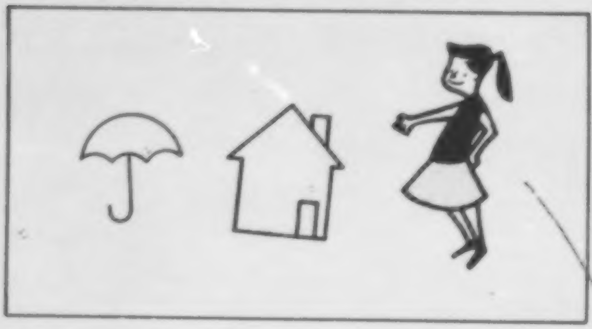






Which sets go with which numbers?



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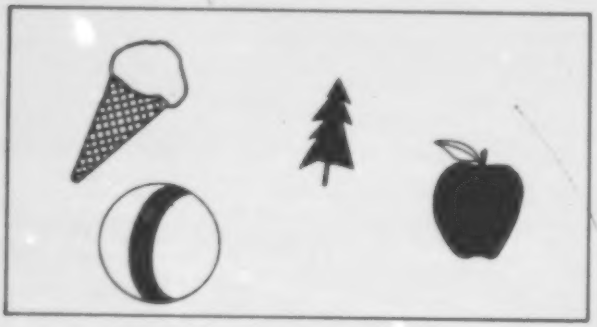

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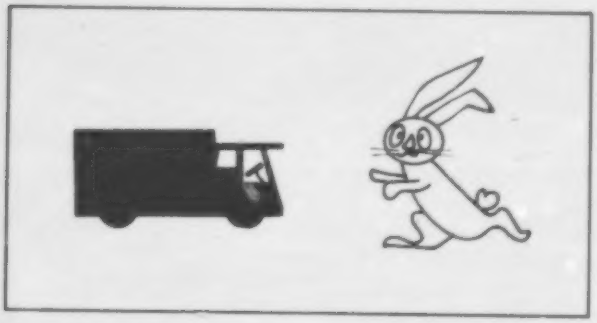

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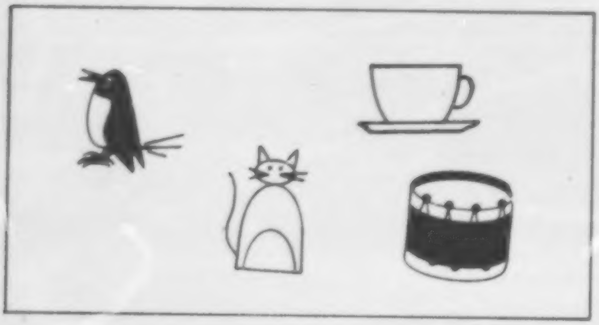
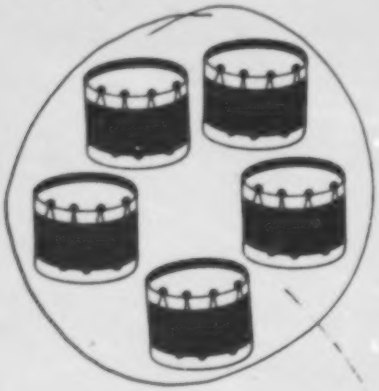
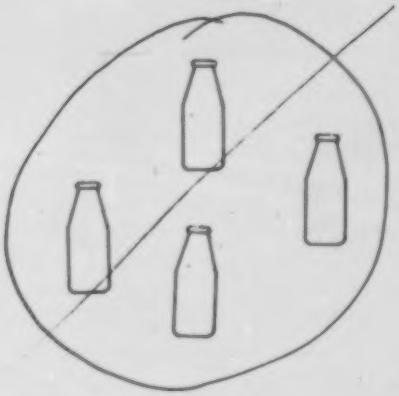
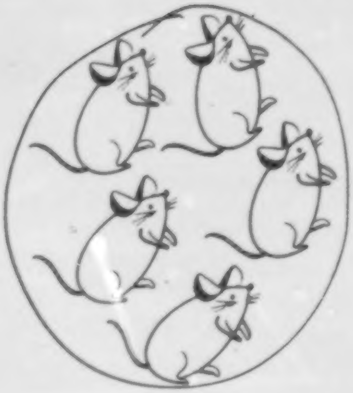


4

Which sets go with 5?







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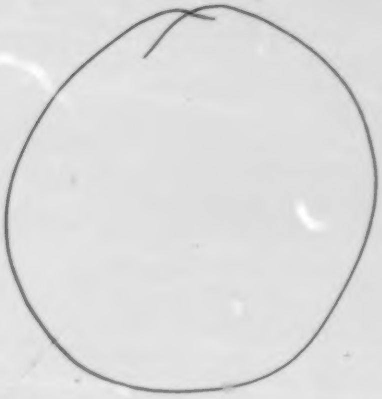
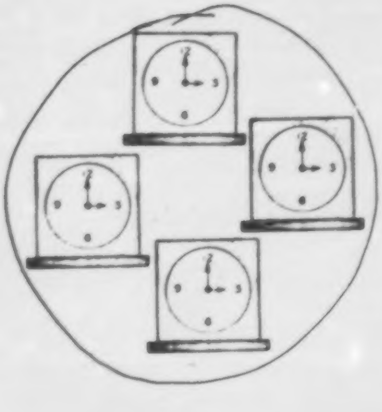
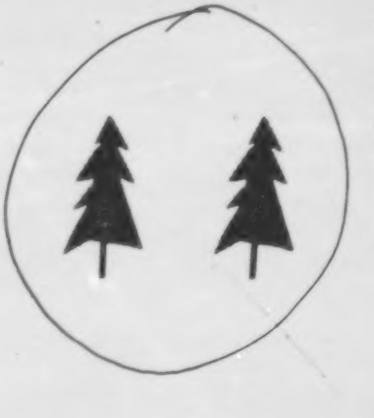
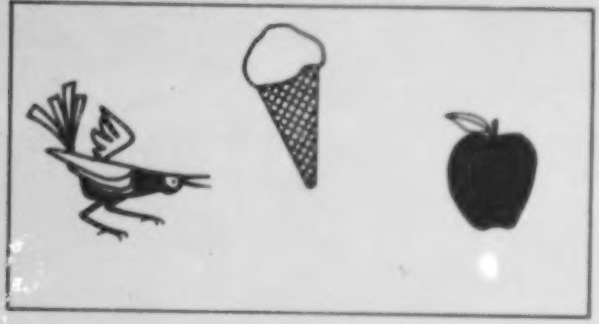






Pair each set with its number name



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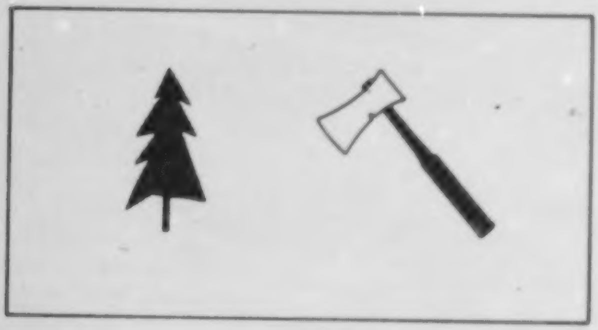

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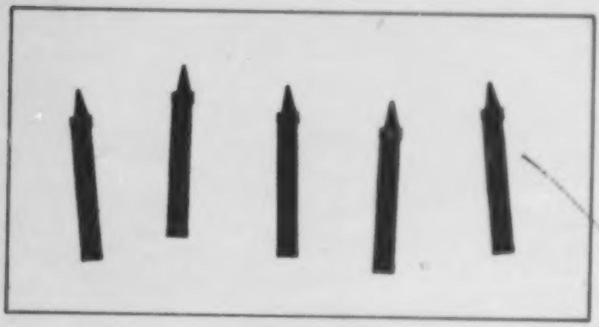

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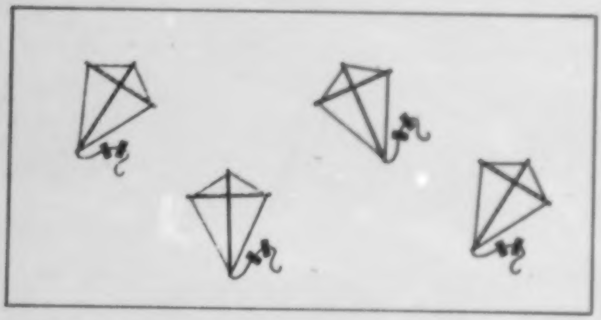


5

USING THE PUPIL'S BOOK, pages 83 - 85

Page 83 is a teaching page. Display Page 83 in front of the class making sure each child has his book open to the correct page. Read the title, pointing to the words as you read. Now, direct the children's attention to the box at the top of the page and ask,

What is the number of this set? (Two.)
Look at the column of three numerals to the right of the set.

Point to the column so the pupils know what column you are talking about. Say,

Do you see the numeral which names the number of this set? (Yes.)
Please ring the numeral which means two.

Have the students trace over the dotted marks. To give the students more practice in reading numerals you can instruct the pupils to read the other two numerals (four and zero) and mark out these numerals with an X. Follow the same procedure through the next two sets.

Pages 84 and 85 are for the pupils to do on their own. Ask the children to ring the numeral that names the number of the set.

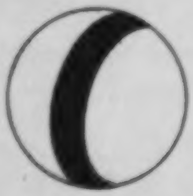
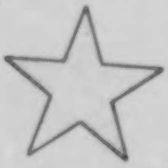
- Give each child a set of 15 blocks or other set objects. Ask them to make sets of 1, 2, 3, 4, and 5 members. After checking to be certain that everyone has ordered the sets correctly, give each child a set of numeral cards 0 - 5. Tell them to place the numeral that names the number of the set below each set.

Is 3 greater than 2? (Yes.)
How do you know that is true? (A set of 3 has more members than a set of 2.)
Suppose someone didn't believe you. How could you use the sets on your desk to show them? (You can see by pairing that a set of 3 has 1 more member than a set of 2.)
Are there any other sets on your desk that have more members than a set of 2? (4 and 5.)
We say that 3, 4, and 5 are greater than 2.

Repeat the same process using the numbers 4, 1, and 3; finally 0 and 5. Hopefully, some of the children will sense that if you locate the sets with the number property named by the numbers you can determine which has more members and hence which number is greater. Those children who seem to be able to do this should be permitted to remove the set objects and use only the numeral cards.

Teaching Page

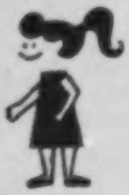
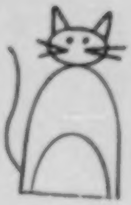
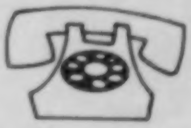
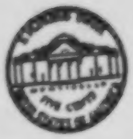
Ring the correct numeral

2

4

0

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3

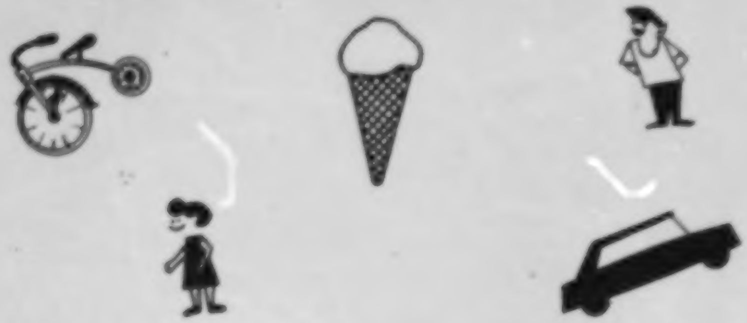
4

3

1

0

Ring the correct numeral



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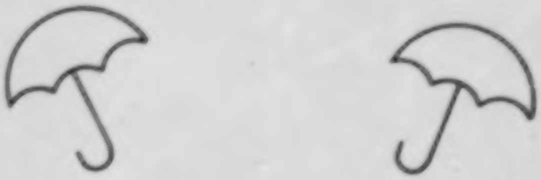
3

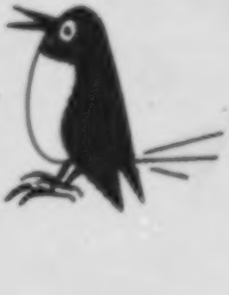
[✓ 5]



1

[✓ 2]

3

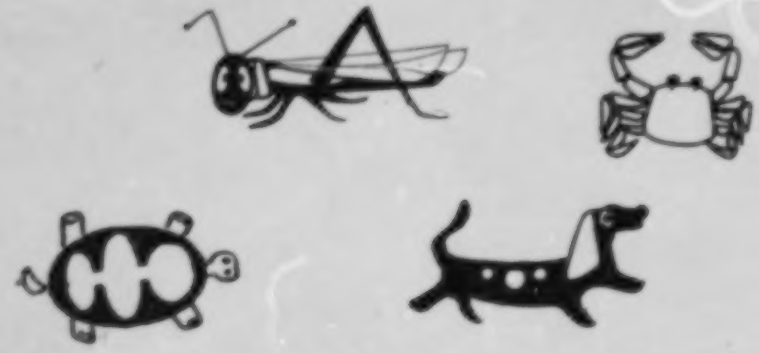


2

[✓ 1]

4

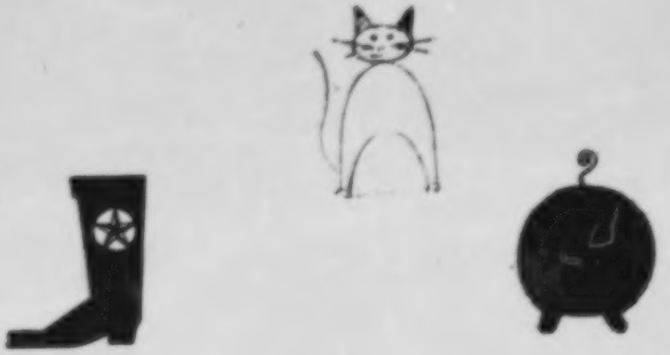
Ring the correct numeral



3

[✓ 4]

5



0

1

[✓ 3]

2

1

[✓ 0]

USING THE PUPIL'S BOOK, pages 86 - 87

Page 86 is a teaching page. Display Page 86 in front of the class. Be sure that each child has his book open to the proper page. Read the title, pointing to the words as you read. Point to the numeral 3 in the upper left-hand corner. Ask the students,

What number does this symbol name? (Three.)

Now point to the numeral 2 which is to the right of 3.

What number does this symbol name? (Two.)
Is 2 greater than 3? (No.)
Let us mark out 2 with an X because 2 is not greater than 3.

Tell students to trace over the dotted X. Now, point to the next numeral to the right.

What number does this numeral name? (Five.)
Is 5 greater than 3. (Yes.)
Let us ring the numeral 5 because it is greater than 3.

Tell students to trace over the dotted ring. Proceed in the same manner through the page.

Page 87 is for independent work.

Pages 88-89:

Use Page 88 as a teaching page. The directions are the same as for Pages 86 - 87, except that the children should ring the numerals which represent numbers which are less than the given number, and mark out the ones which represent numbers which are greater.

Teaching Page

Which numbers are greater?

3	~~2~~	[✓ 5]	[✓ 4]
4	~~0~~	[✓ 5]	~~1~~
1	[✓ 2]	[✓ 4]	~~0~~
5	~~4~~	~~0~~	~~1~~
0	[✓ 3]	[✓ 5]	[✓ 2]
2	~~0~~	[✓ 4]	~~1~~

Which numbers are greater?

2	[✓ 3]	~~0~~	~~4~~
0	[✓ 5]	[✓ 3]	[✓ 1]
5	~~0~~	~~2~~	~~3~~
3	~~1~~	[✓ 5]	[✓ 4]
1	[✓ 2]	~~0~~	[✓ 3]
4	~~0~~	[✓ 5]	~~2~~

Teaching Page

Which numbers are less?

3	[✓ 2]	~~5~~	~~4~~
4	[✓ 0]	~~5~~	[✓ 1]
1	~~2~~	~~4~~	[✓ 0]
5	[✓ 4]	[✓ 0]	[✓ 1]
0	~~3~~	~~5~~	~~2~~
2	[✓ 0]	~~4~~	[✓ 1]

Which numbers are less?

2	[✓ 1]	~~4~~	[✓ 0]
4	[✓ 2]	~~5~~	[✓ 3]
1	~~3~~	[✓ 0]	~~5~~
5	[✓ 2]	[✓ 1]	[✓ 3]
0	~~4~~	~~2~~	~~1~~
3	~~5~~	[✓ 0]	[✓ 2]

USING THE PUPIL'S BOOK, pages 90 - 91

Page 90 is a teaching page. Ask the students to write the number of the set in the space provided. Always give instructions as clearly as possible making sure each pupil has his book open to the correct page. It is also helpful to read the title of each page.

Pages 92 - 99:

These pages provide practice in identifying the numerals which name the numbers six through nine. Each set of two pages requires the same directions as Pages 75 - 82.

Pages 100 - 102:

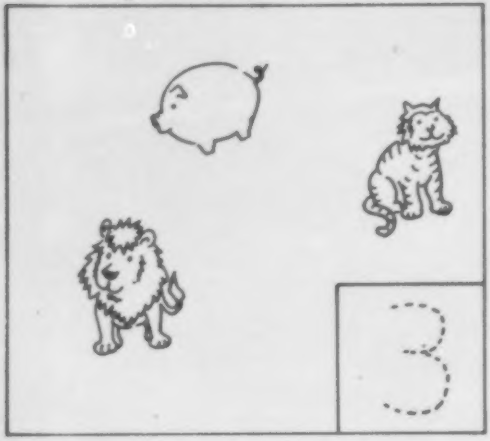
The directions for these pages are the same as those for Pages 83 - 85.

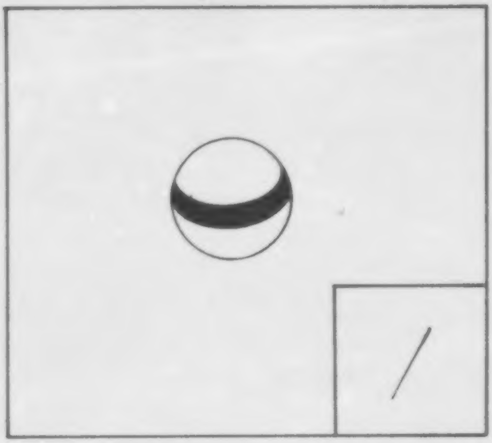
Page 103:

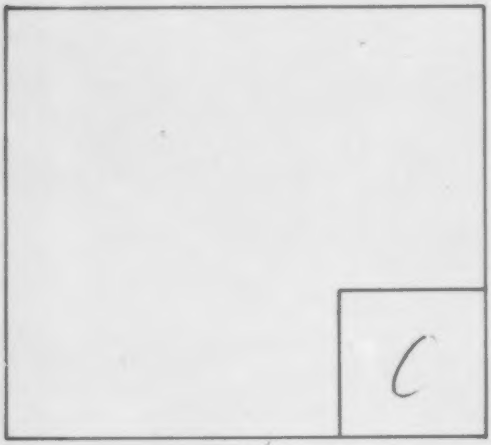
After the usual preliminary procedures, ask the children to draw a line from the set to the numeral that names the number of the set.

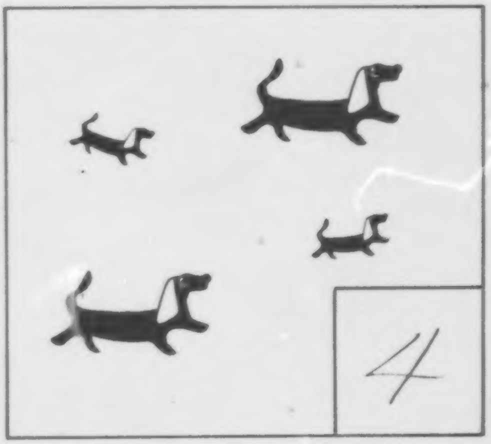
Teaching Page

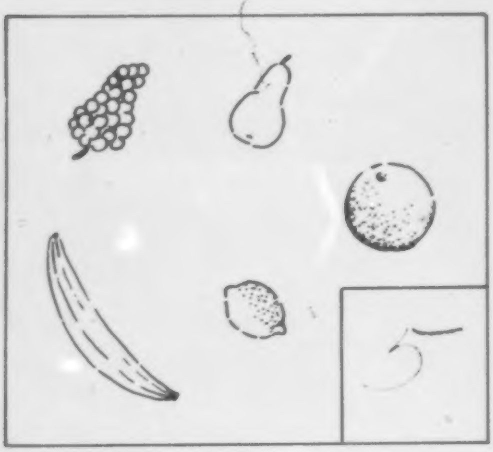
How many in each set?

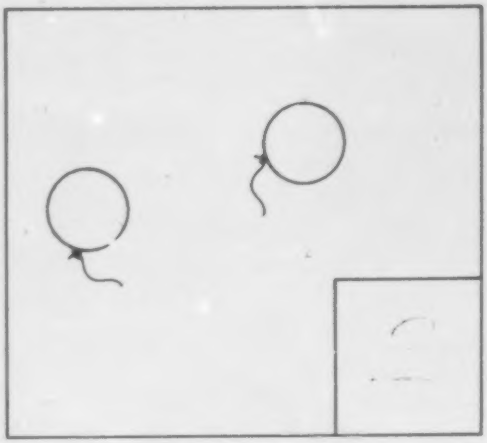




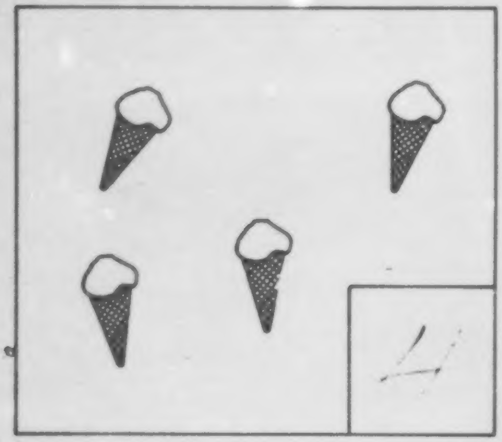








How many in each s ?



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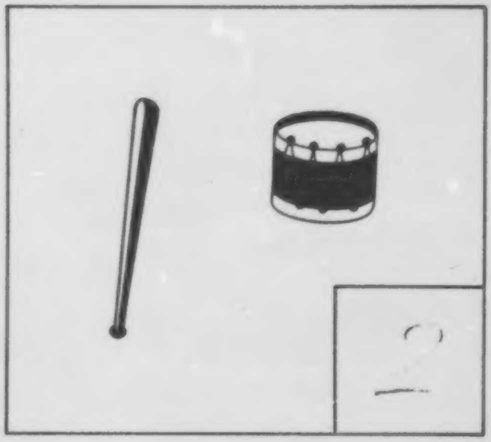

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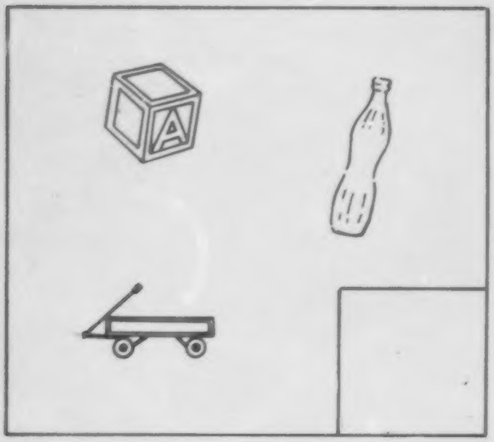

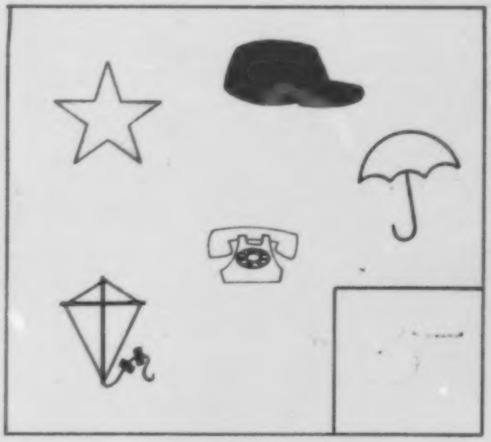

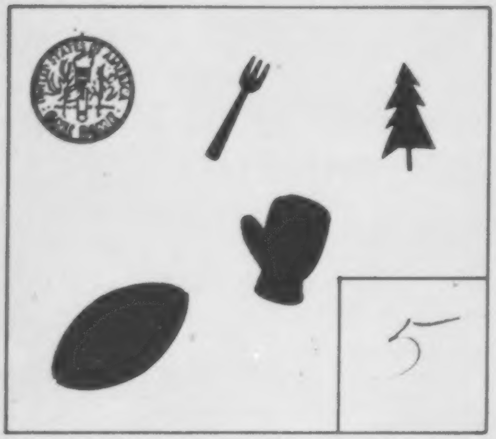

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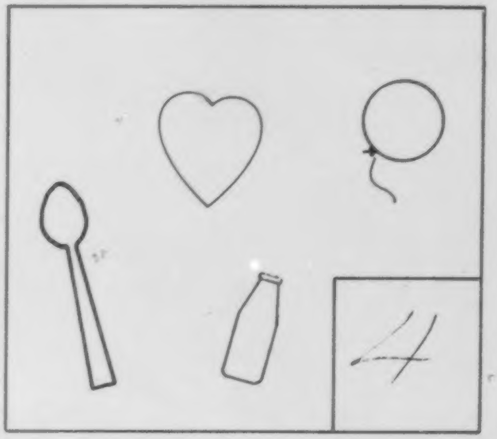

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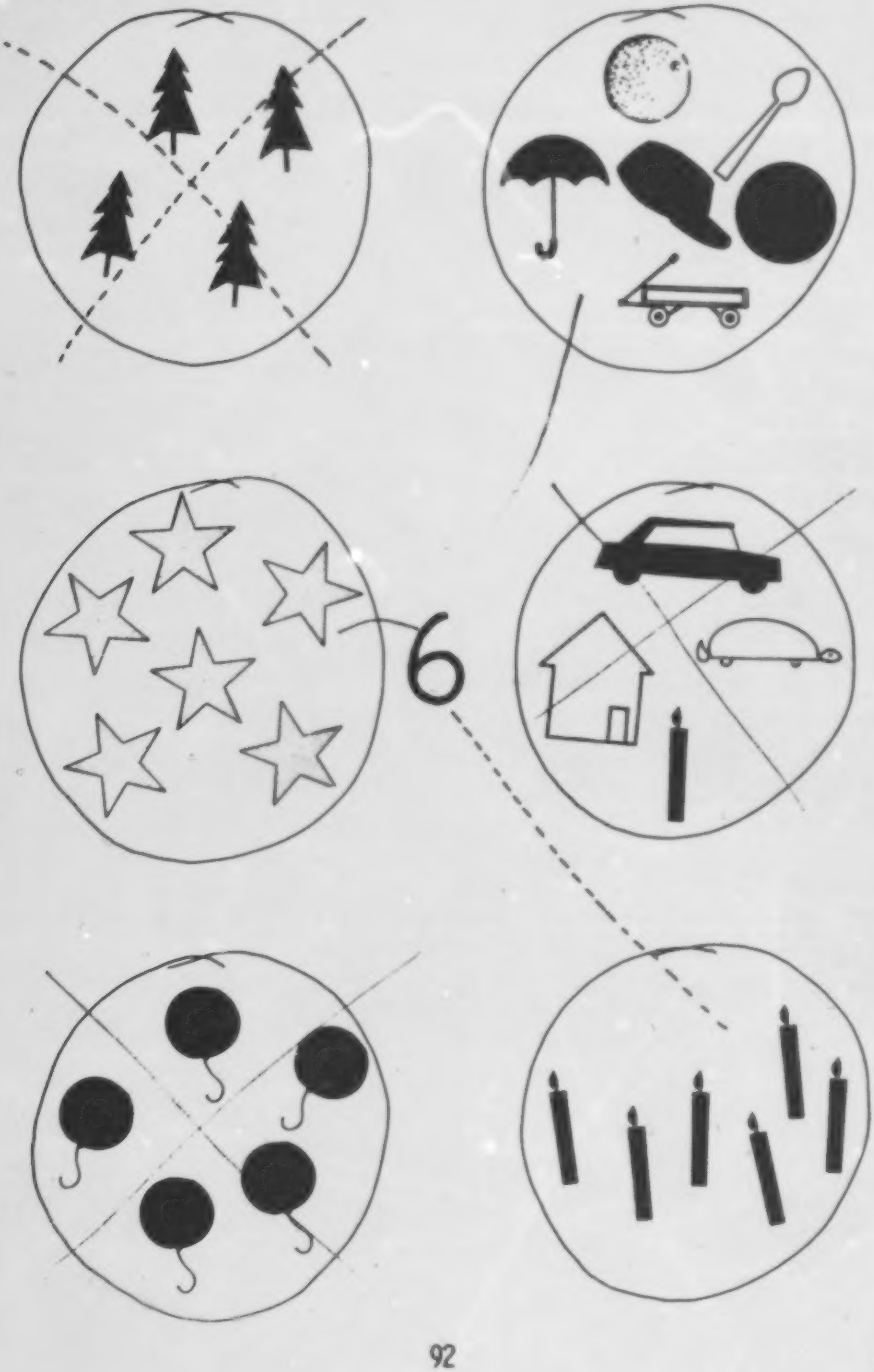
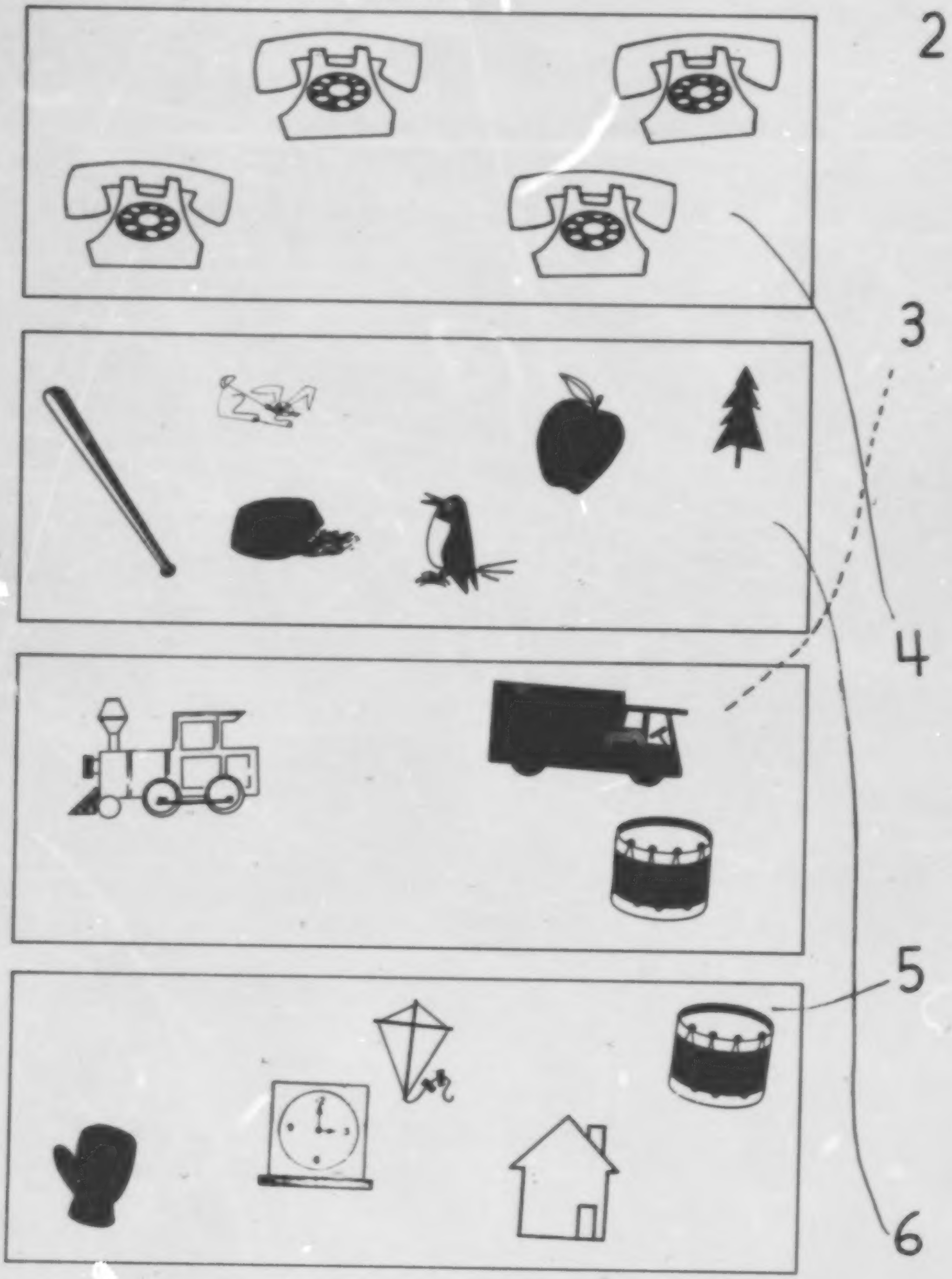
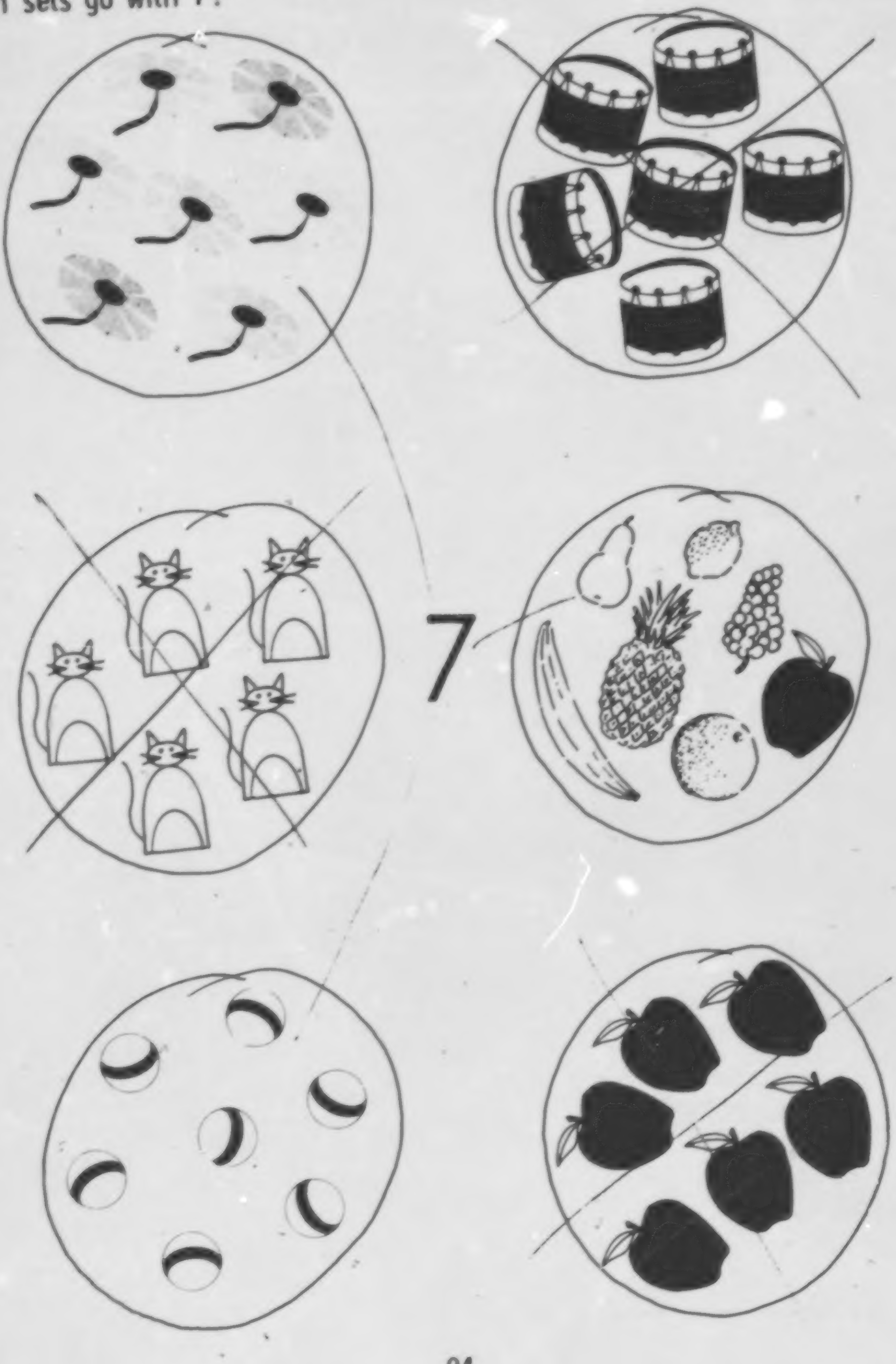
Which sets go with 6?



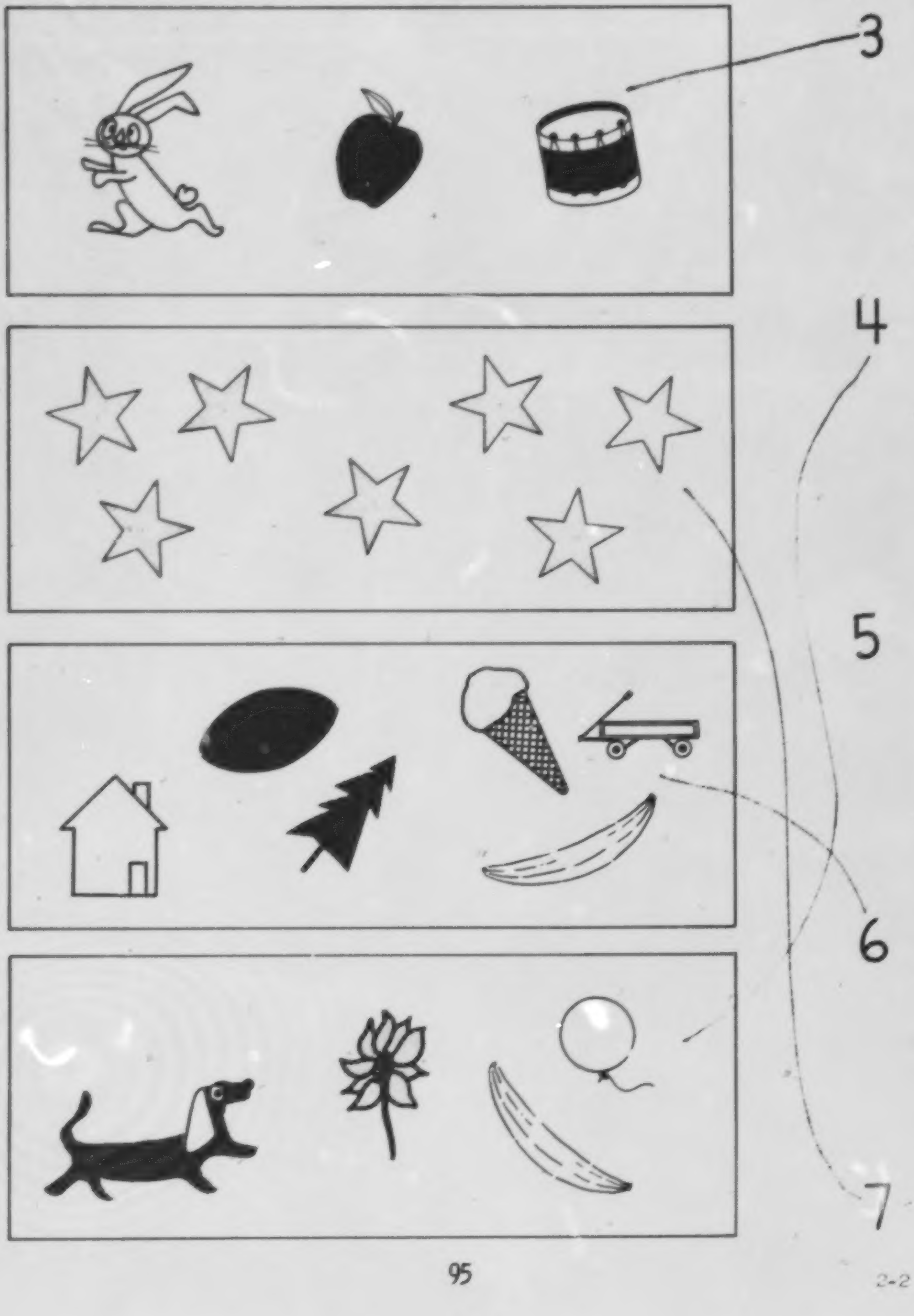
Which sets go with which numbers?



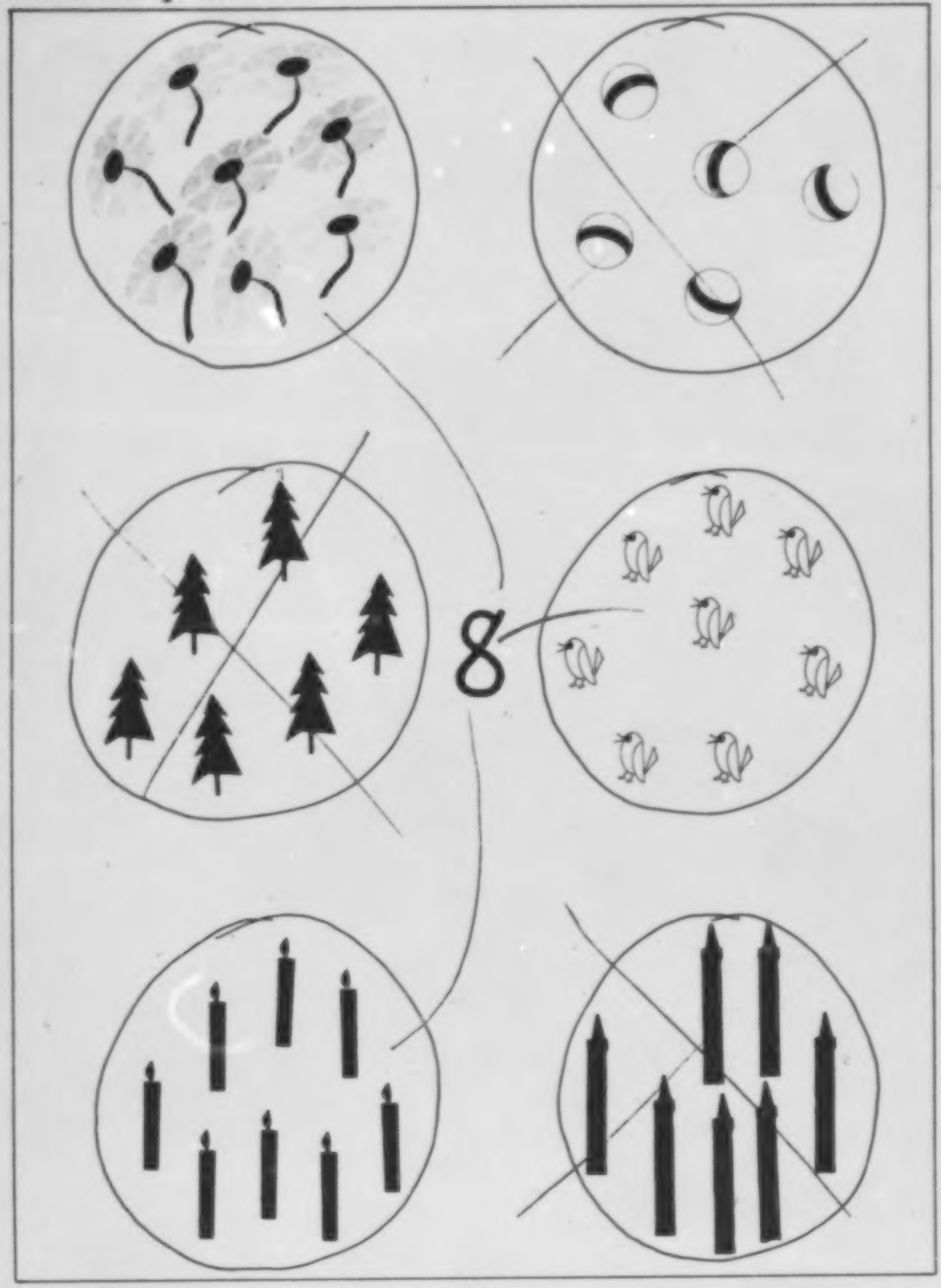
Which sets go with 7?



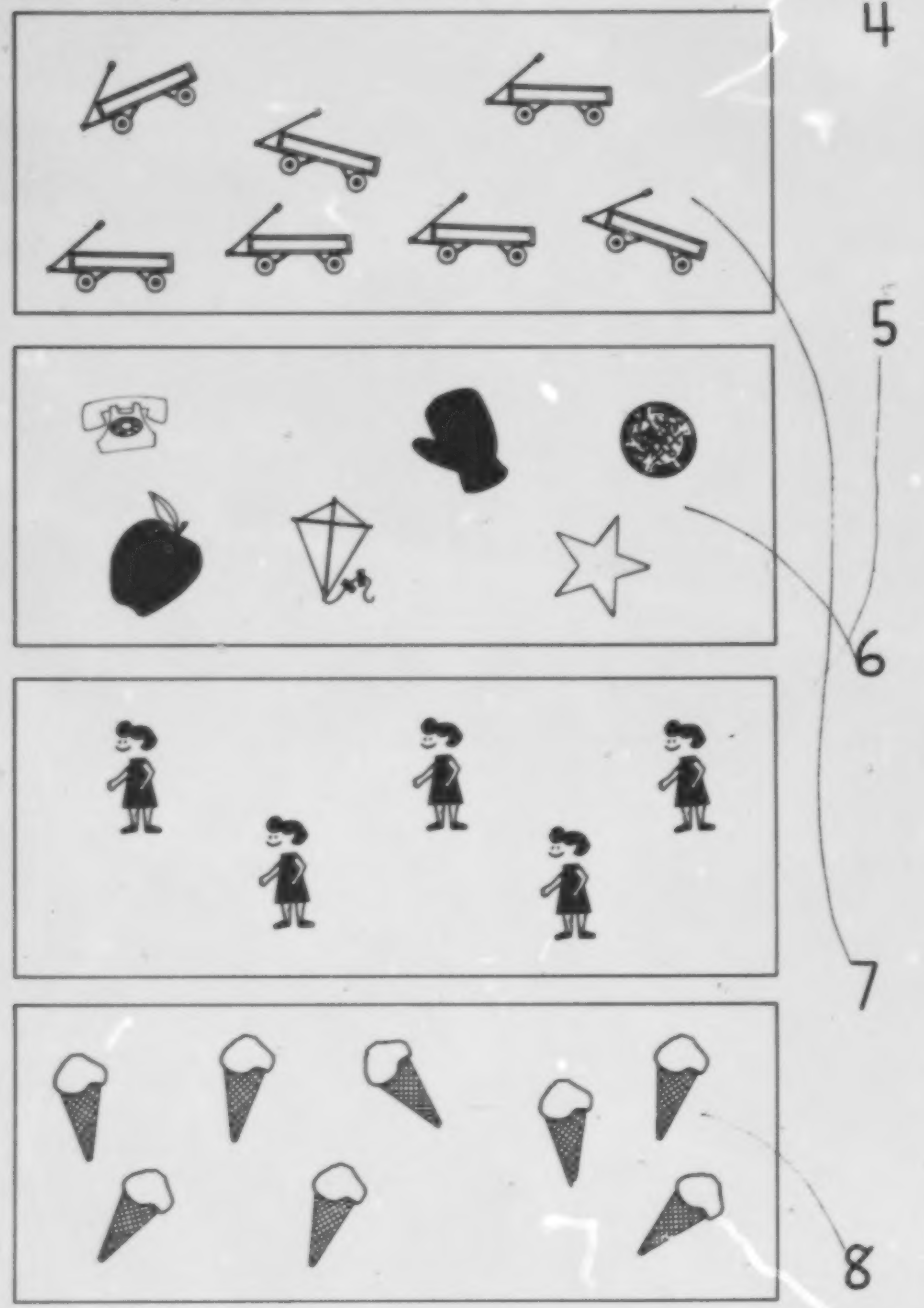
Which sets go with which numbers?



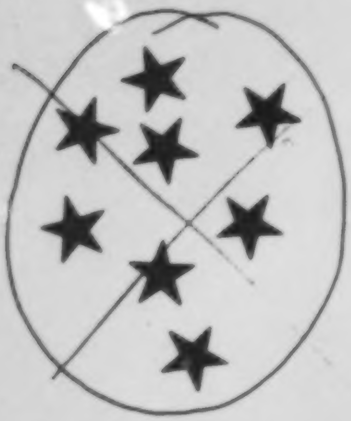
Which sets go with 8?

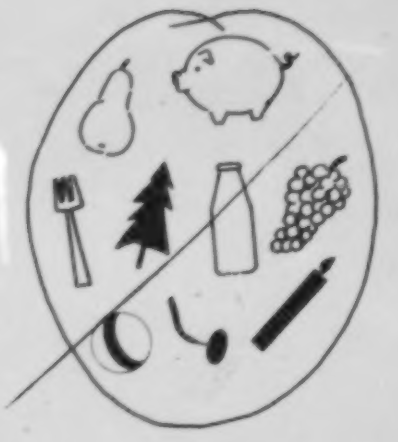


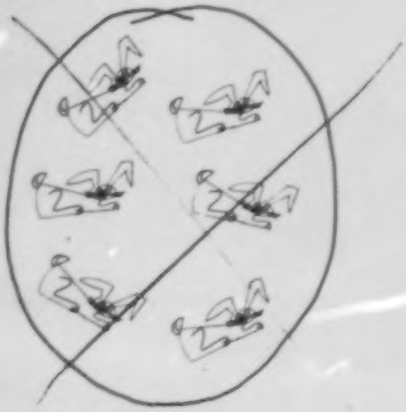
Which sets go with which numbers?



Which sets go with 9?







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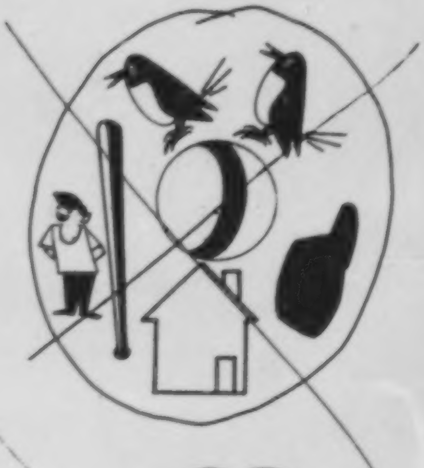
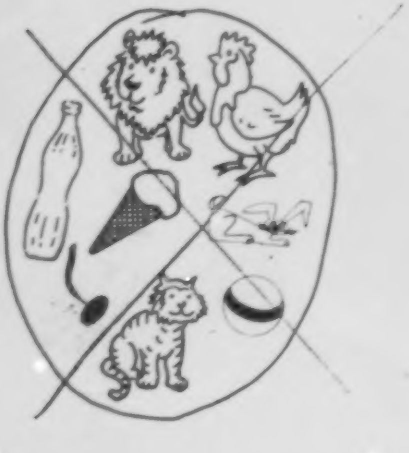
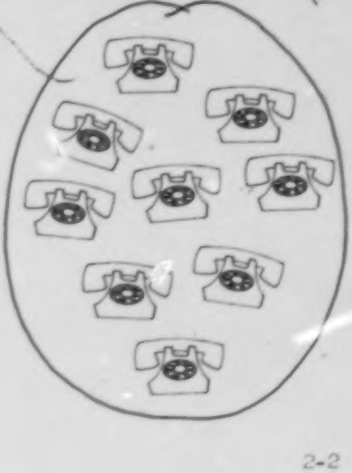
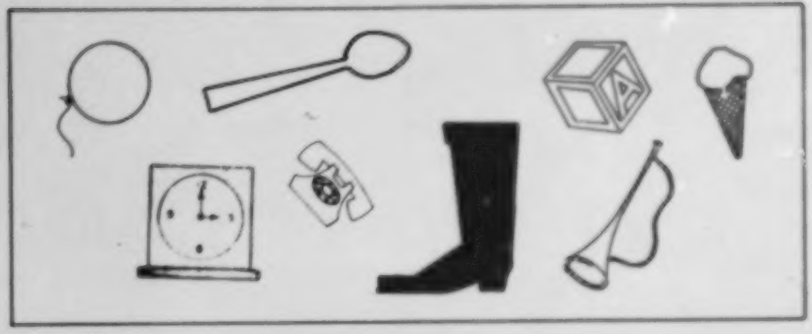






Which sets go with which numbers?



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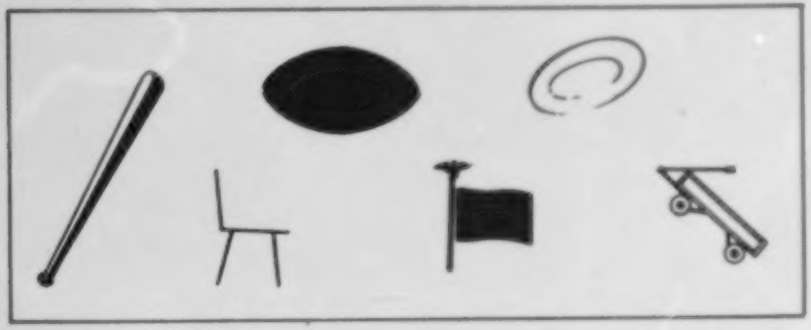

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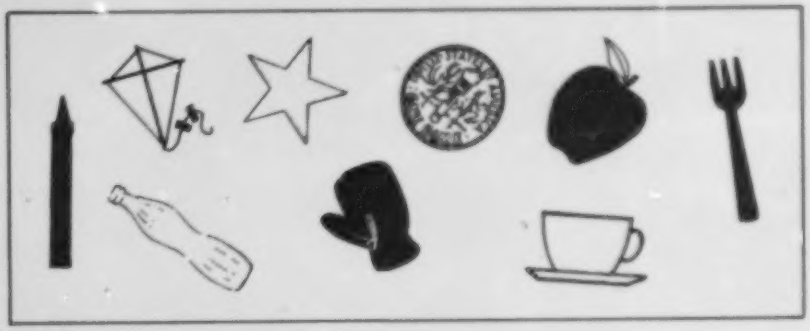

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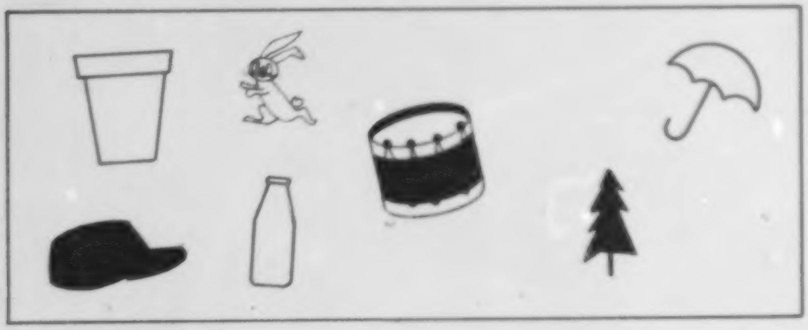
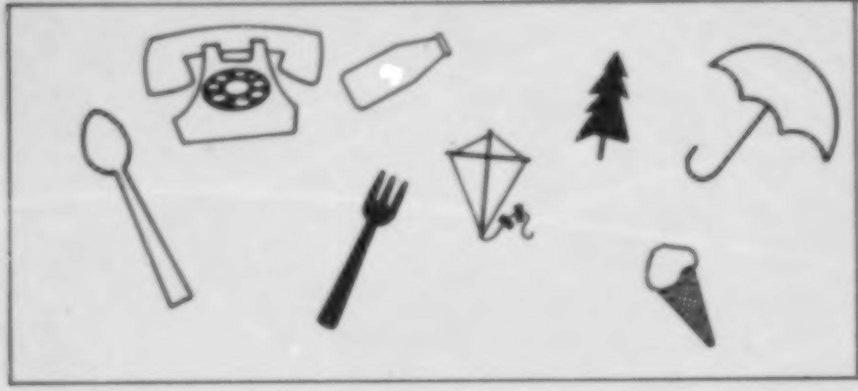


9

Ring the correct numeral



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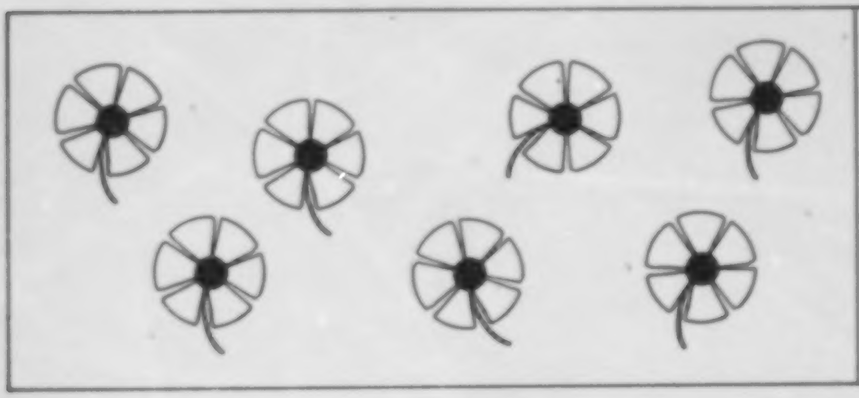
7

[✓ 8]



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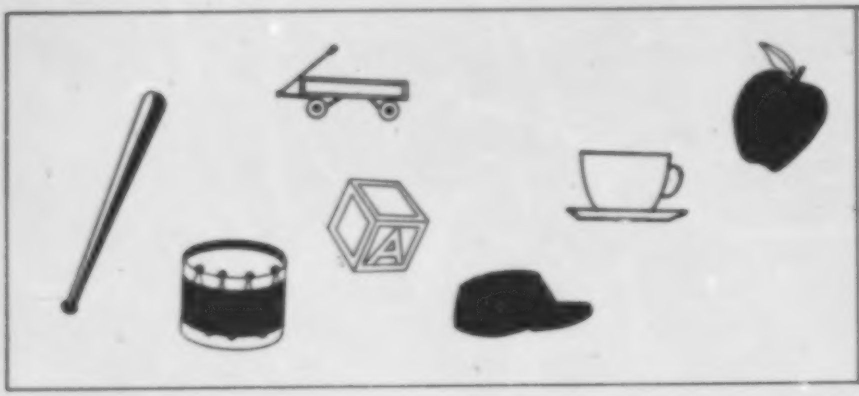

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[✓ 7]

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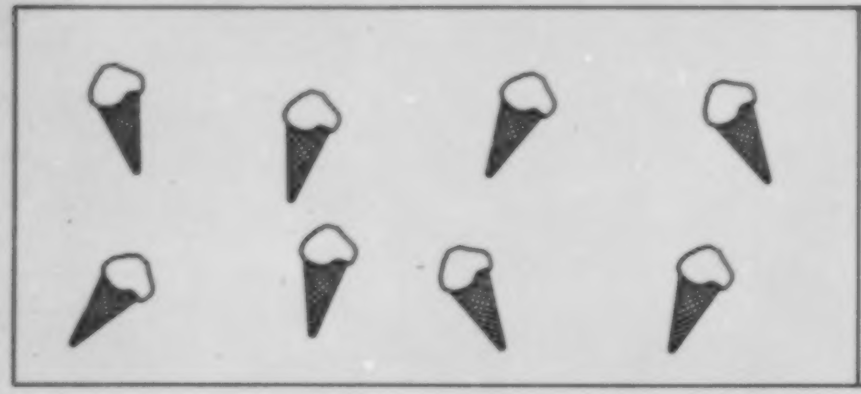
Ring the correct numeral



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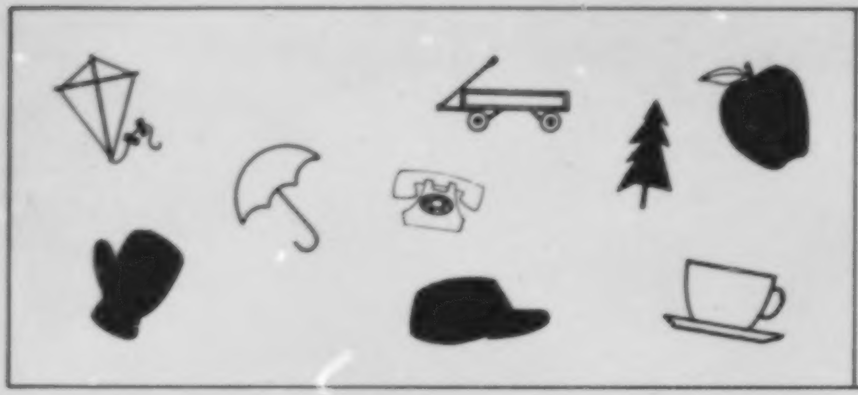

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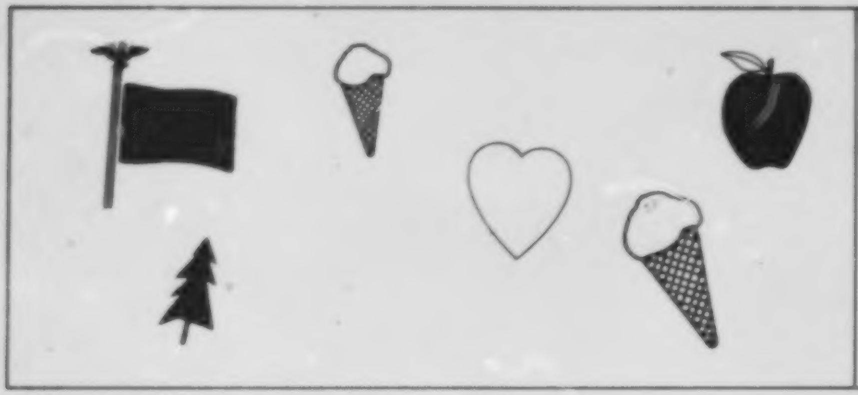

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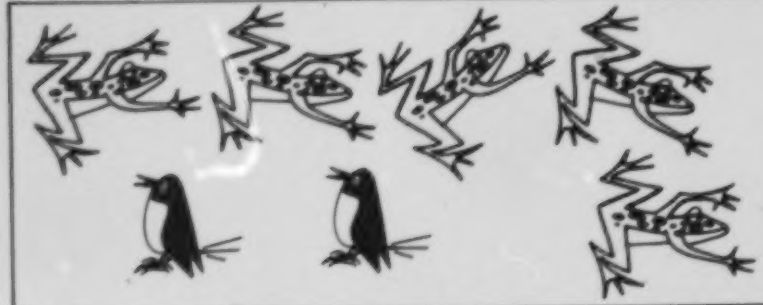
Ring the correct numeral



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[✓ 7]

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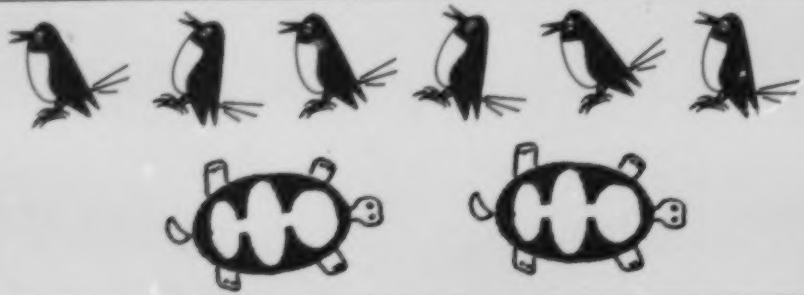

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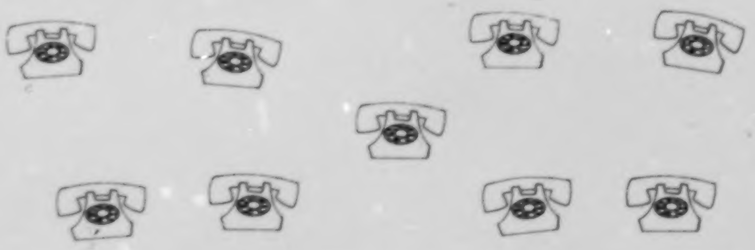

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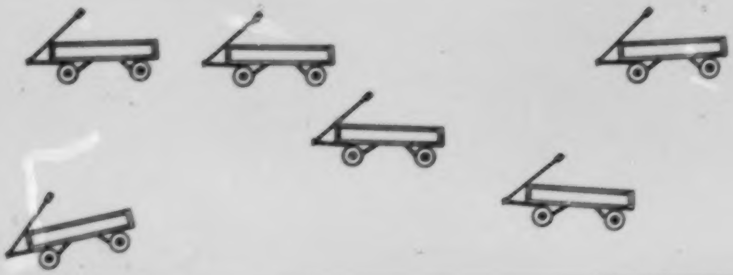

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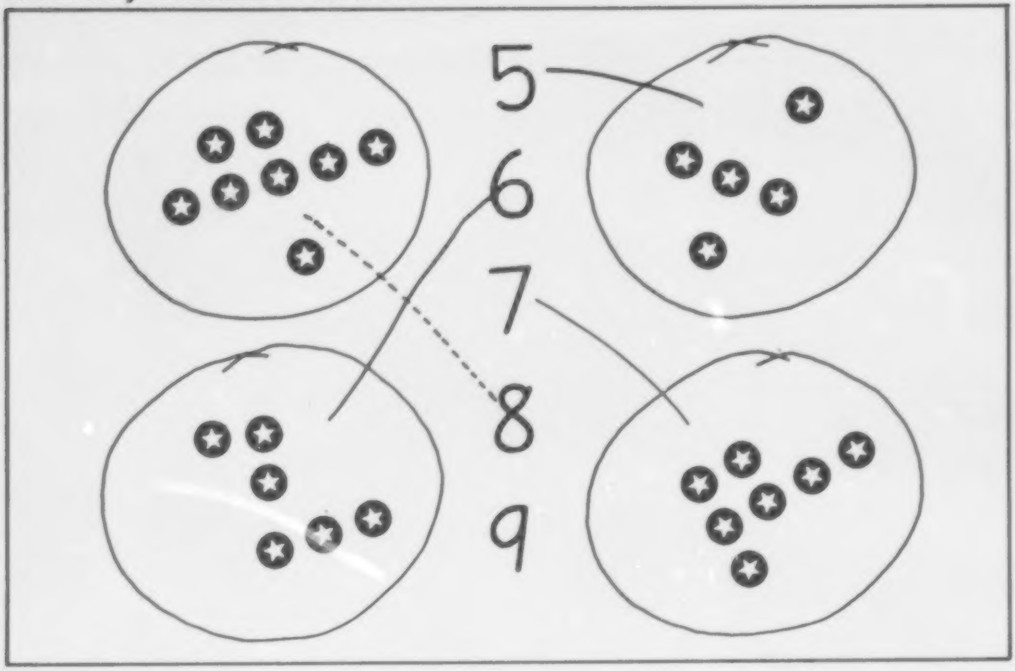
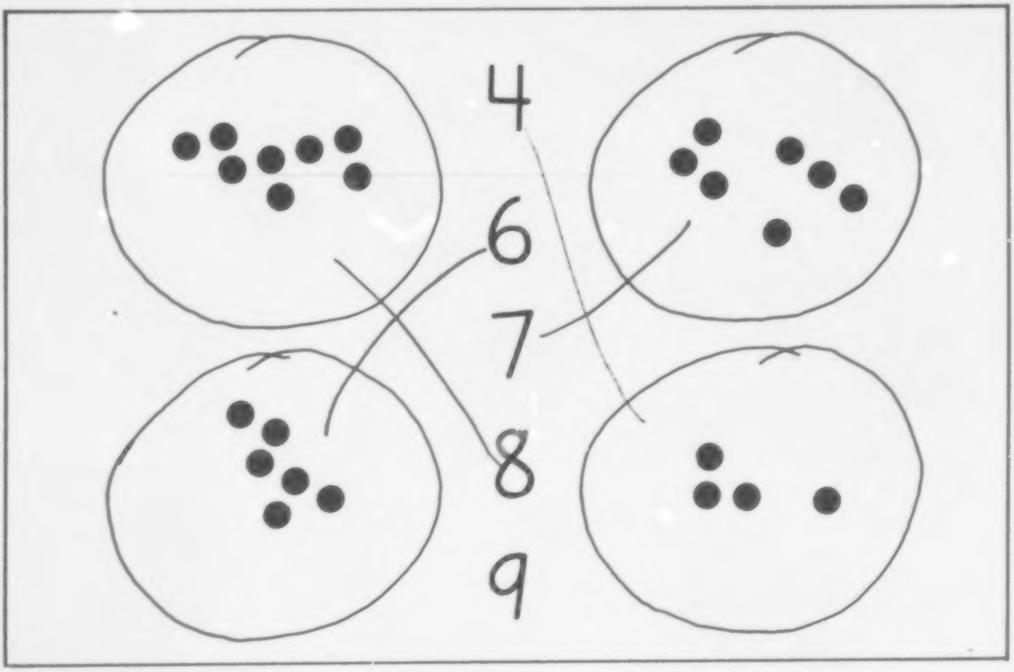
6

1

How many members in each set?





PART II - THE NUMERALS 6 THROUGH 9

SUGGESTED PROCEDURES:

PRE-BOOK ACTIVITIES:

Repeat the same procedures for introducing the numerals 6, 7, 8 and 9 that you used with the numerals 0 through 5. If you introduce a new numeral on each succeeding day, 2 pages have been provided in the pupil's book for review and the recognition of the new symbol. Careful attention should be given to an examination of the numerals. Similarities and differences should be noted. While 8 is easily recognized, 6 and 9 are often confused and the 7 when written by children is often as Γ .

After all the numerals 0 - 9 have been introduced, you should keep a display on the chalkboard, flannel board, or bulletin board of the following sort:

```
                           *
                        *  *
                     *  *  *
                  *  *  *  *
               *  *  *  *  *
            *  *  *  *  *  *
         *  *  *  *  *  *  *
      *  *  *  *  *  *  *  *
   *  *  *  *  *  *  *  *  *
0  1  2  3  4  5  6  7  8  9
```

For these lessons, it is essential for children to be able to count on from a given number. When part of a set is recognized children should count the additional members of the set without starting again at one. Encourage this kind of counting (partial counting) but do not assume that all children will master this concept readily.

Distribute 7 to 10 objects to each child. Have child arrange the set on his desk so that members of the set are not touching. Without moving any of the objects, ask children to frame 3 members of the set then count the rest of the set. Frame 3, 4, or 2 members of the set on the desk. Increase or decrease the number of objects with which a child is working and repeat the work. Continue the work by asking a child to frame the number of members which he can name without counting. Then ask the child to push those objects aside and count the rest of the set.

USING THE PUPIL'S BOOK, pages 104 - 105

Ask the children to count the number of members in the set and write the numeral that names the number.

- Use the flannel board display of ordered sets and numerals to help children identify numbers greater than a given one. The same type of procedure suggested for numbers less than 6 may be used in this situation. Then give the children numeral cards 0 - 9. After checking to be certain that each child has ordered the numerals correctly, the pupils may use the ordered numerals to help them determine which numbers are greater than a given one.

Page 106:

After the preliminary procedures, point to the box in the upper left hand corner. Direct the children's attention to the numeral in the upper-right of the box and ask,

What number does this numeral name? (Four.)

Now point to the numeral in the lower-left of the box and ask,

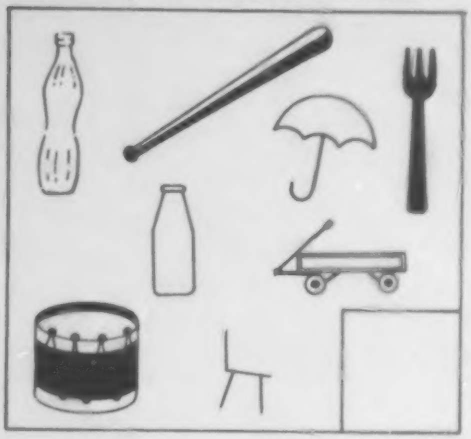
What number does this numeral name? (Two.)
Which number is greater, two or four? (Four.)
Mark the numeral 4 with a ring.

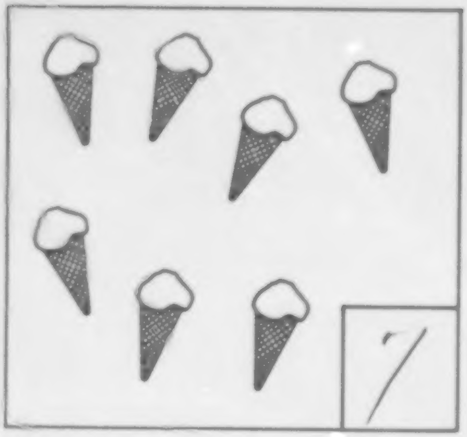
Use the same procedure for each of the boxes in the top row. By this time, most of the students will know the procedure and they can finish the rest of the page on their own.

Page 107:

The instructions are the same as for Page 106, except that now the students are asked to ring the lesser number.

How many members in each set?





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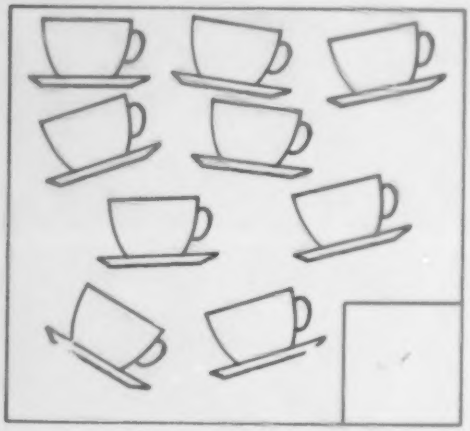
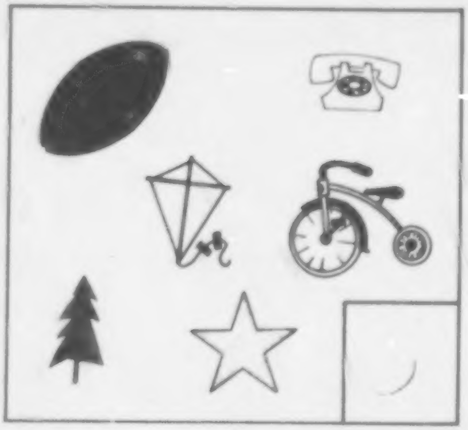
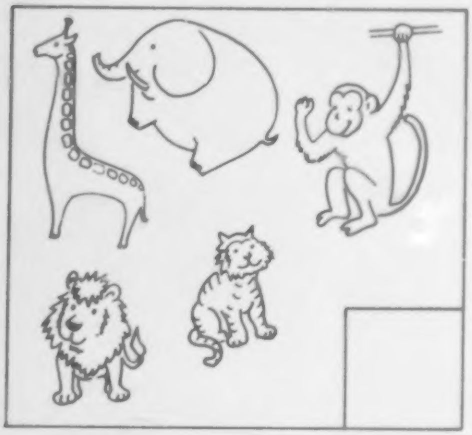
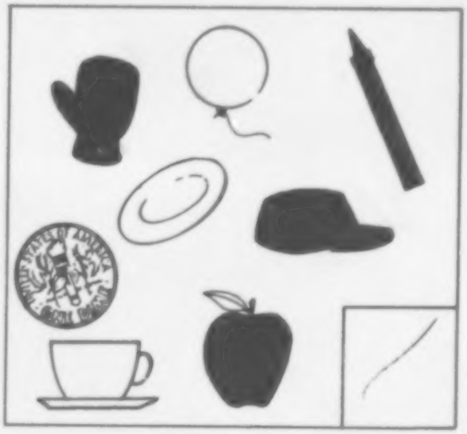
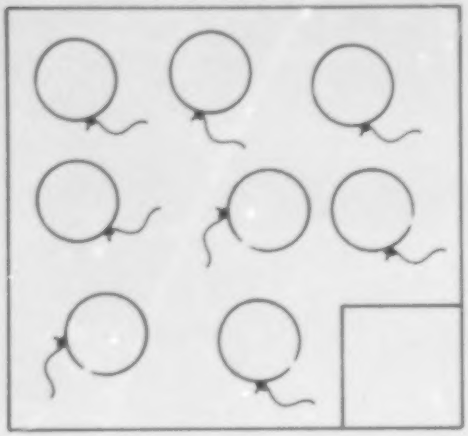


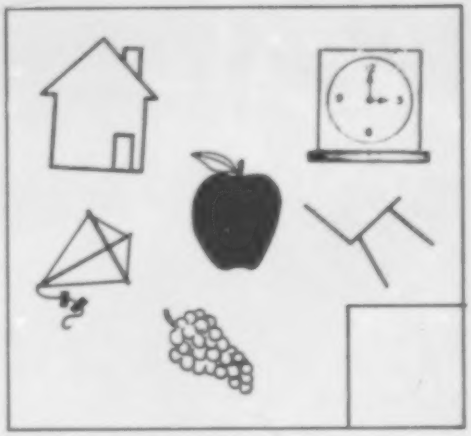


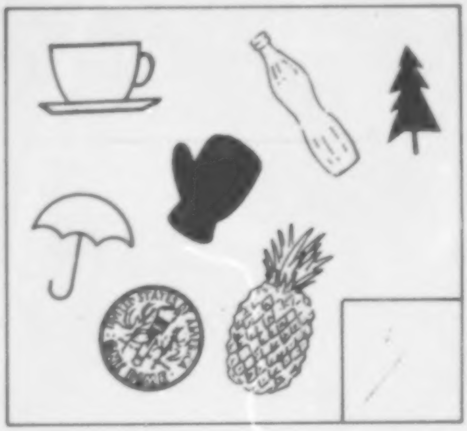


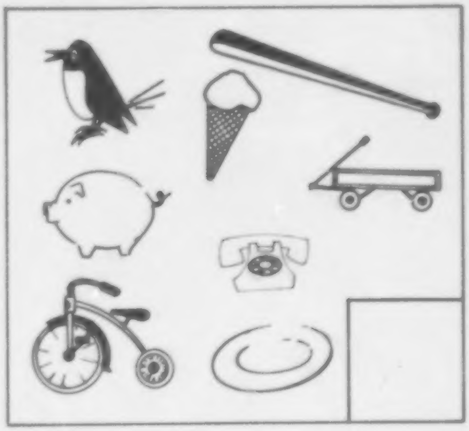


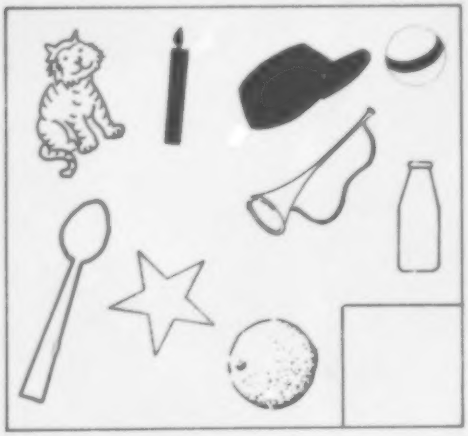
How many members in each set?

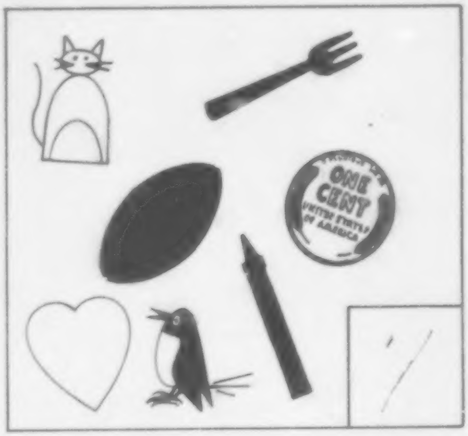








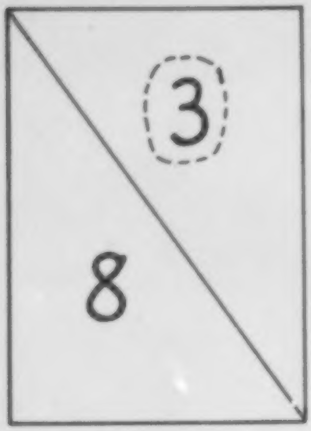


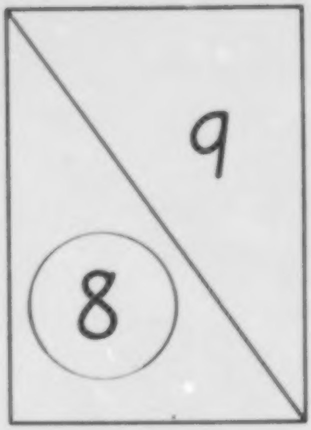

ONE CENT
UNITED STATES OF AMERICA

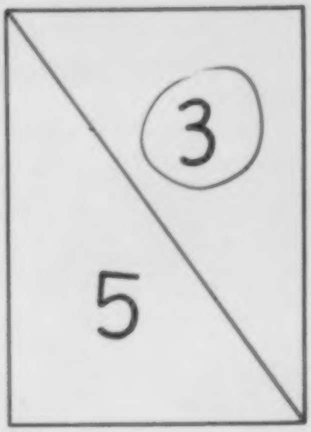
Which number is greater?

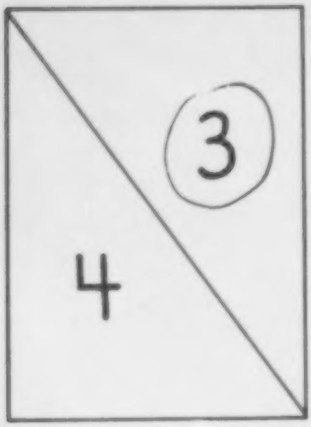
[✓ 4] 2	1 [✓ 3]	[✓ 9] 5
0 [✓ 2]	6 [✓ 7]	[✓ 3] 2
[✓ 9] 0	5 [✓ 7]	[✓ 9] 2

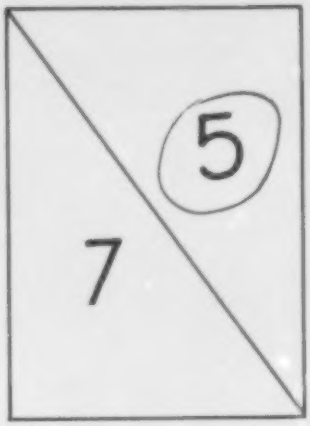
Which number is less?

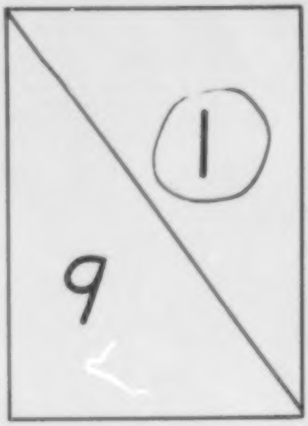


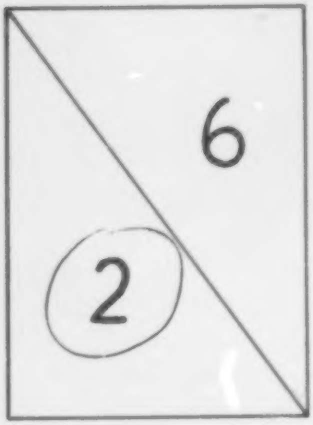


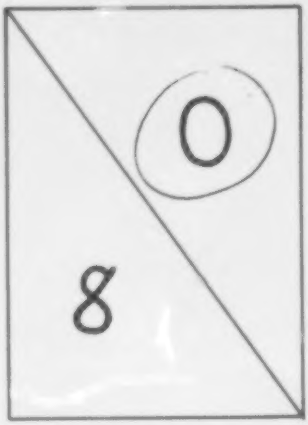


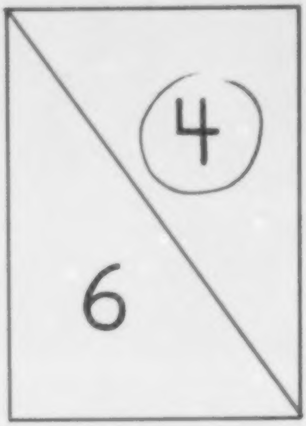












PART III - THE WORDS, "ZERO, ONE, TWO, ..., TEN"

Children should learn to recognize the words, zero, one, two, three, four, ..., ten as names for numbers just as they recognize the numerals 0, 1, 2, 3, 4, etc. Some children who are capable of understanding the concepts which are presented in this chapter and can learn the numerals without difficulty may not yet be ready to recognize the words. It is not intended that mastery of words precedes continuation of the book but that a start be made so that when they are used there will be some familiarity with the words. You should expect a level of mastery which seems appropriate for the ability of your children.

Since the teaching of these words is closely related to reading, you should use techniques and activities which you use in your reading program to teach these words. You will want the children to associate the written word with the spoken word, with the set, and with the numeral. Where the words are used on pupil pages you will find that they are used with the numeral. Cards with numerals on one side and the words on the back, and cards with pictures of a set of objects on one side and the word on the back provide for individual or small group practice.

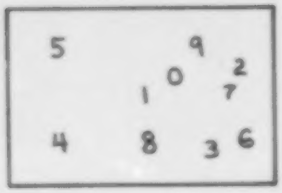
FURTHER ACTIVITIES:

1. Give each child a set of numeral cards. Name a number and ask the children to show the correct numeral for that number.

2. Give each child a set of numeral cards. Show sets of objects on the flannel board or on the chalk board, describe sets in the room or show pictures of sets, and have each child show the correct numeral card.

3. Pass out sheets of paper, each with a numeral written on it. Ask the children to draw a set with that many members.

4. Give each child a piece of newsprint or construction paper. Tell the children to make each numeral 0 1, 2, 3, 4, 5, 6, 7, 8, 9 on the page. The numerals should be scattered all over the page and each child's page will probably be different. Display a set of objects on the flannel board. Ask the children to find the numeral which tells the number of members in the set. They "show" the numeral by framing it with their hands.


5
9
2
0
1
7
4
8
3
6

5. The first three activities may be repeated using word cards instead of numeral cards.

2-3. THE NUMBER LINE

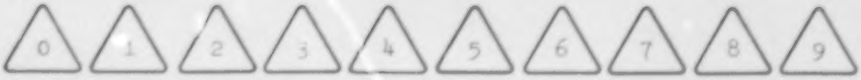
OBJECTIVE: To develop the idea of the number line.

VOCABULARY: Point, line.

BACKGROUND NOTE:

This lesson introduces the number line. Eventually we shall use the number line for addition, for subtraction, and to introduce rational numbers. In this lesson, the principal use of the number line is to reinforce the children's concept of inequality.

MATERIALS: Strong string, paper clips, a printed number line and two sets of numeral cards shaped as shown:



SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Have one set of numeral cards in random order on the chalk ledge. Ask a child to come to the front of the room. Tell children, "Today we are going to jump and count the jumps." Mark, on the floor, a starting point.

Susie, you stand at our starting point. (To Susie:)
How many jumps have you taken from the starting point? (None.)
What number tells how many jumps you have taken? (0.)

Place the zero card by the child's feet with the point toward her feet. Tell her to take a jump and stop.

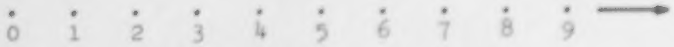
How many jumps from the starting point have you taken now? (1.)

Have a child choose the numeral card and place it at Susie's feet. Continue with the same procedure until 9 jumps have been taken. Ask the child to try to take jumps of the same size (distance). Keep emphasizing that the numeral card tells how many jumps from the starting point.

Would we have to stop with 9 if we had more cards? (No. We could go on and on.)

Remove the numeral cards from the floor and tell the children to imagine that jumps had been taken on the chalkboard. Have a child indicate the starting point and nine jumps. Make a dot to show each of these points. Ask what numeral to write for the point that show the starting point. (0.)

Continue to number the points through 5. Have children look at unnumbered points, and ask one to come up and put the pointer on the point where he thinks the next jump would be. Ask if points could continue to be made in this direction. Draw an arrow to show in what direction we would go if nine jumps are taken. (We begin with dots and an arrow and gradually build the picture of the number line.)



Try to establish for the children, using further experiences if necessary, the picture of equally spaced dots extending indefinitely to the right, with each dot associated with a number. Repeat that the numeral names the number of jumps from the starting point.

USING THE PUPIL'S BOOK, page 108

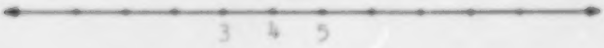
Ask the children to draw a line from the numeral to the point under which it should be placed.

- A day or two later, have two children, one at each end, pull the piece of string tightly. Ask children to pretend to watch a frog jumping across a tight rope. Suggest a spot on the string (to the left as the class sees it) where it begins, and ask what numeral to use at this point. Clip the zero card to the string (using a paper clip at the top of the card).

Have a child come up and stand by the string at that point, take a single jump and "stop" there. Place numerals after children tell you which one for each jump.

Ask children to imagine a long, long tight rope on which many jumps can be taken. Suggest that it be thought of as a line of numbered jumps, all of the same length, or a "number line".

Dispense with the string and draw a line on the board. Mark points and label as shown



Instead of using a tight rope for our number line, let's use a line on the board.
How is this number line different from the one we made on the tight rope? (It doesn't have all the numerals. It doesn't show the first jump.)

Ask a child to use the pointer to show another point, to tell what numeral to write there, and to tell why he decided on that numeral. (Name the number of jumps from the starting point.) Have children notice that although not all the numerals are written, they know where others must go because of the ones already written.

Have the other points named and write the numerals (not beyond 9 however). Emphasize that 0 is the starting point, and that each numeral tells how many jumps from the starting point.

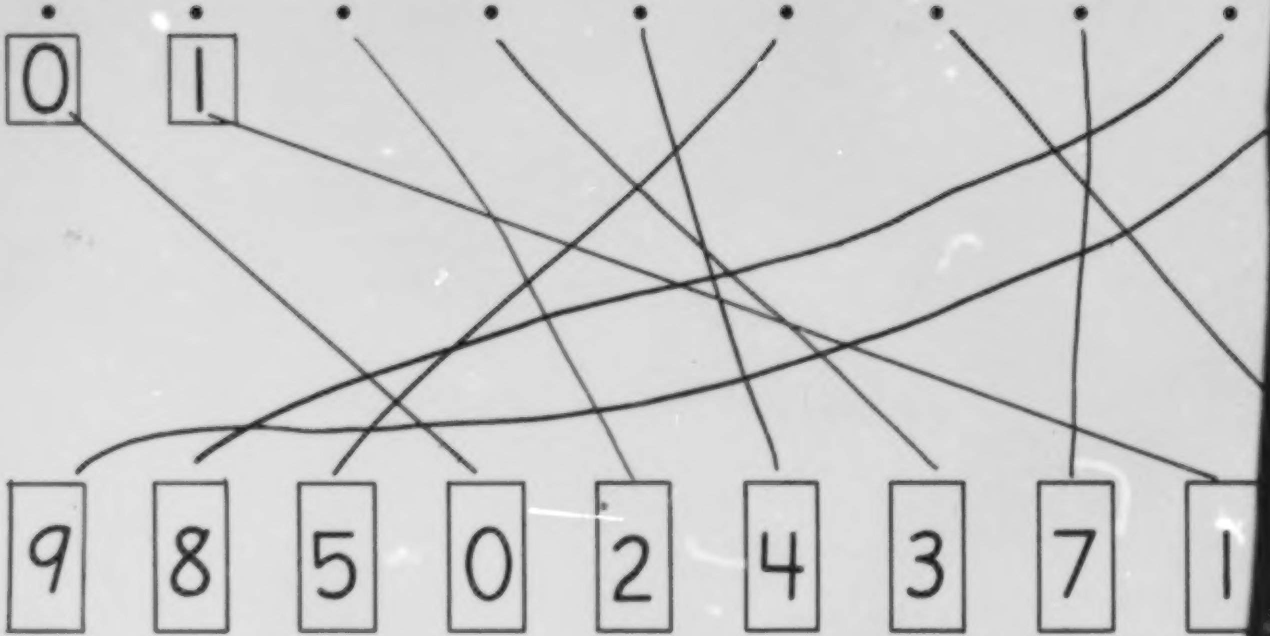
Use the line for children to point to the numeral before "5", the numeral after "1", before "1", etc.

Now help the children to understand the relation between inequality of numbers and order on the number line.

Is 5 greater than 3? (Yes.)
Is 6 greater than 3? (Yes.)
Is 7 greater than 3? (Yes.)
Is 8 greater than 3? (Yes.)
People have agreed that as we move to the right (indicate what you mean by right by moving your hand in that direction along the number line) on the number line that the numbers become greater.

Place your left hand below 3 and your right hand below 5. Ask the children how they can tell from looking at the number line that 5 is greater than 3. If the children say that 3 is before or in front of 5 or that 5 is after or behind 3, agree. Then comment that 5 is greater than 3 since 5 is to the right of 3 on the number line. Even though children can tell their right hand from their left, it does not mean that this concept will necessarily transfer to the number line situation. A left hand is always a left hand and a right hand is always a right hand. On the number line 5 is to the left of 7 but to the right of 3. Be content to accept the children's own terminology as long as they convey the correct meaning. Agree, and then repeat the correct language. Eventually the children will associate this language with the concept and use it.

Where do the numerals belong?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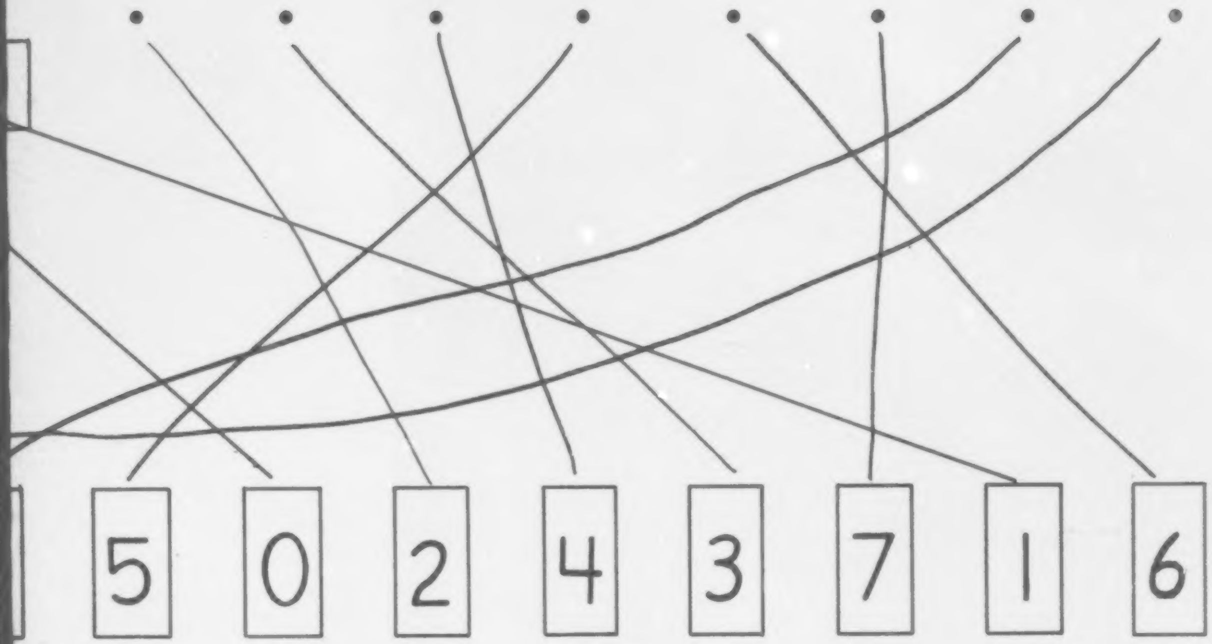
1

numerals belong?



Page 109:

Ask the children to write the numeral that names the points given on the number line.

Page 110:

The children are to mark all the numerals that name numbers greater than the number named in the box at the left. The number line at the top of the page is provided to assist those children who need this visualization.

- Repeat the procedure used to introduce the concept of "greater than" in connection with the number line. This time find the number or numbers which are less than a given one.

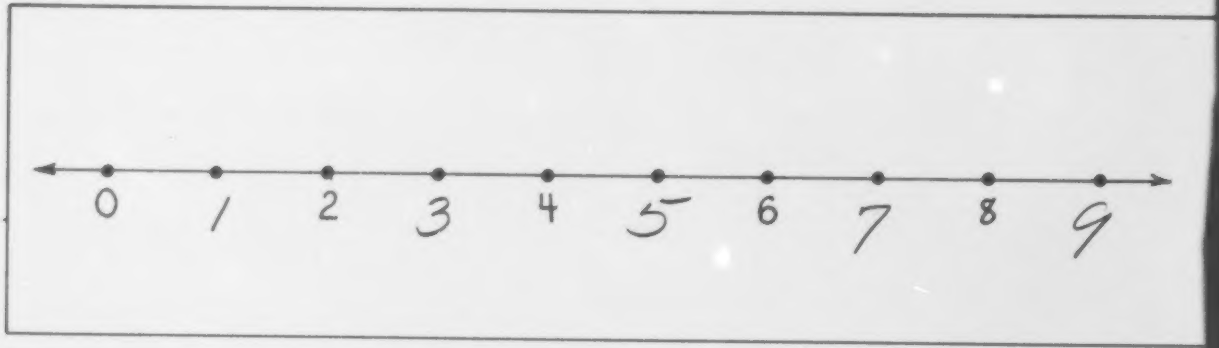
Page 111:

Ask the children to mark all the numerals that name numbers less than the number named in the box at the left.

FURTHER ACTIVITIES:

1. Point to a given numeral on the number line and ask for a number or numbers greater than the given one. Practice of this kind will develop the idea that all numerals after or to the right of 5, for example, name the numbers greater than 5.
2. Point to a given numeral on the number line and ask for a number or numbers less than the given one. The idea that all the numerals before or to the left of 5, for example, name the numbers less than 5 will develop.
3. Provide a number line (0 - 9) for each child. Attach it to the desk with tape. This will be a reference for those children that cannot recognize all the numerals. Children can be asked to point to a numeral that names a number greater than or less than a given one.
4. Provide each child with numeral cards 0 - 9. Name a number and ask the children to hold up a numeral card that names a number greater than or less than the given one.

Name the points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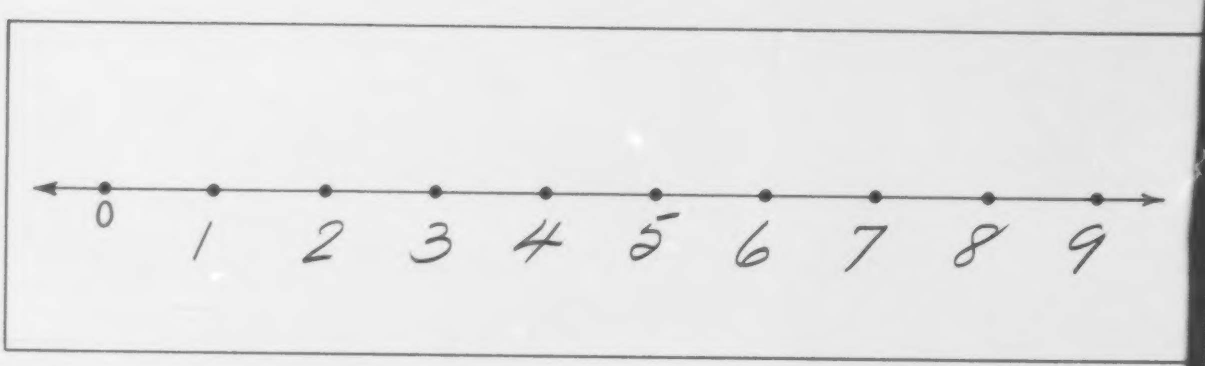
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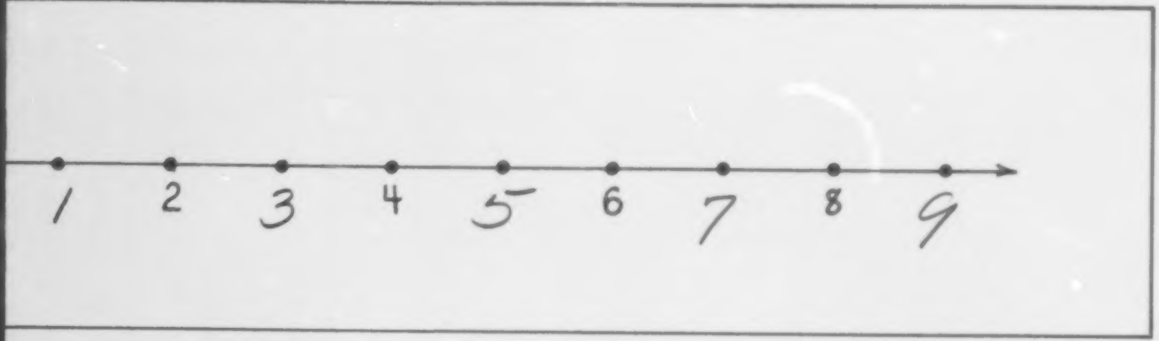
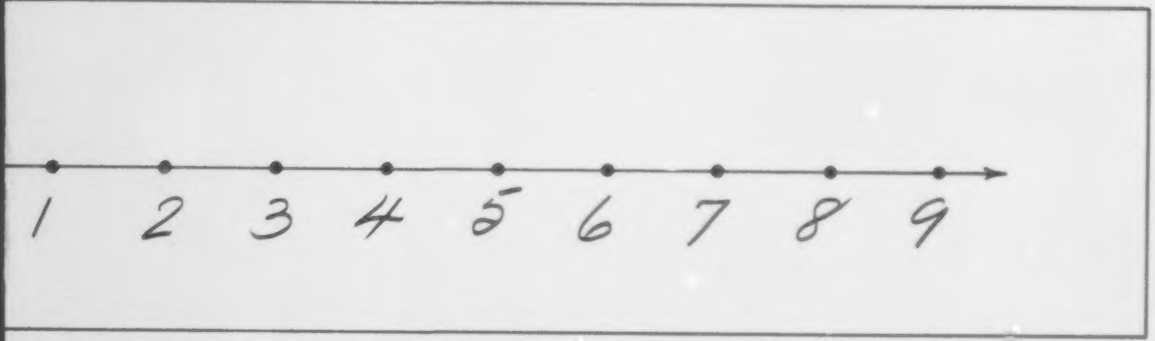
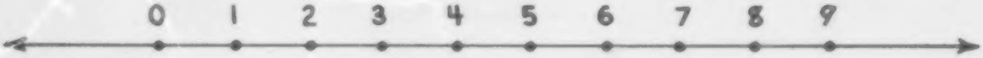
nts





Use the number line


0
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2	[✓ 5]	[✓ 7]	[✓ 3]	0	1
7	3	[✓ 9]	0	[✓ 8]	
4	[✓ 8]	4	1	[✓ 9]	
4	[✓ 6]	2	[✓ 7]	[✓ 5]	

Use the number line

0
1
2
3
4
5
6
7
8
9

7	[✓ 4]	8	[✓ 2]	5
4	[✓ 3]	6	[✓ 0]	9
1	9	6	2	[✓ 0]
9	[✓ 3]	[✓ 7]	[✓ 6]	[✓ 8]

Chapter 3

SETS OF TEN

BACKGROUND

We have been concerned so far with sets of single objects--that is, with bunches of things. In this chapter we reach a slightly higher level of sophistication: we consider sets whose members are themselves sets. We count sets of objects by partitioning them into sets of ten and then counting the sets of ten. We extend our system of numeration by agreeing that, for example, a set which can be partitioned into 3 sets of ten has thirty members.

This short chapter is devoted entirely to sets of ten and it is primarily preparation for the study of place value. In a later chapter, we shall partition a set into equivalent subsets which are not necessarily sets of ten. We then connect the fact that, for example, a set of thirty can be partitioned into 5 sets of 6, with the facts: $5 \times 6 = 30$ and $30 \div 5 = 6$.

3-1. SETS OF TEN

OBJECTIVE: To introduce counting sets of ten members each.

VOCABULARY: (No new words.)

MATERIALS: For teacher--a box with sets of ten disks and other cut-outs for use at the flannel board; large sheet of tagboard or cardboard; counting sticks; rubber bands.

For pupils--sets of materials in the multiples of ten (10 to 100 of each); e.g., blocks, kindergarten beads, buttons, lima beans, spools, sticks, theatre tickets, pegs, and paper clips.

TEACHING NOTE:

The activities described here may be carried on for several days, the time depending on the ability and previous experiences of the children. It should be remembered, however, that the intent is to "open up" an idea rather than to develop full understanding.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Show the children a box containing thirty flannel cut-outs (ten disks and ten each of two other shapes), bunched so that it is necessary to count and sort them. Dump them on a table.

I wonder how many sets of ten are in this box?
Let's see how we can find out.

Pick out one disk and hold it up.

How many disks have I? (One.)

Place that disk on the flannel board. Repeat the procedure, making a row of disks on the flannel board. (As you work, a child may be asked to help pick out the cut-outs with which you are working and hand them to you, one at a time.)

How many disks are there in the set on the flannel board?

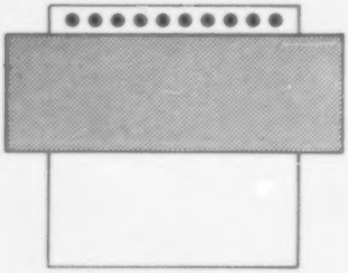
Point to each disk as children count to ten.

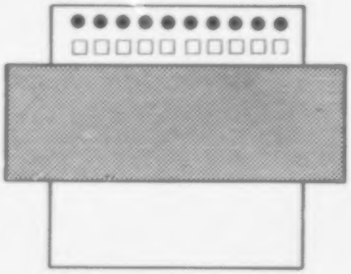
This set of disks on the flannel board has how many members? (Ten.)
Do we have ONE set with ten members? (Yes.)
Let's see if we can use the set of ten disks to find how many trees (or other shapes you have selected) we have.

Place ten trees in a row beneath the disks so that the one-to-one correspondence is obvious.

Is the set of trees equivalent to the set of disks? (Yes.)
What is the number of the set of disks? (Ten.)
Of the set of trees? (Ten.)
How many sets of ten cut-outs do we have on the flannel board? (Two.)

Hold tagboard to cover the sets, exposing one row, then two, while children count with you: one set of ten, two sets of ten.





Place another set of ten cut-outs beneath the trees.

How many sets of ten cut-outs have we now? (Three.)

Again cover and expose rows while children count: one set of ten, two sets of ten, three sets of ten. Tell the children that we can say "one ten", "two tens", "three tens", and not say as many words when we count. Let them repeat the counting: "one ten", and so on.

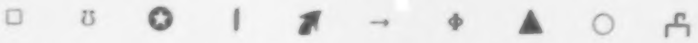
How many sets of ten cut-outs did we have in the box? (Three.)

Have children count ten sticks as you bundle them together. Ask how many are in the bundle; put a rubber band around it. Continue in like manner with ten sets (bundles) of ten sticks, asking questions similar to those related to the sets of cut-outs on the flannel board.

Have sets of ten blocks counted and stacked in "tens", questioning the children as the work proceeds as to how many tens there are after each stack is completed.

OTHER EXPERIENCES WITH SETS OF TEN

Paper clips may be strung together as sets of ten to lead to the idea that ten ones and one ten are names for the same number. Sets with a variety of members as well as with similar objects should be grouped into sets of ten, e.g.



A chart may be made on the chalkboard with blanks in which children can record the number of sets of ten that have been counted.

_____ tens   _____ tens

_____ tens   _____ tens

The preceding experiences have had considrable guidance by the teacher. The next step, and an important one, is to give each child a set of materials to count into sets of ten. Paste sticks or tongue depressors are excellent set objects. It is not necessary that everyone have the same number of sticks. Some children will laboriously count sets of ten and place them in piles on the desk while others may count one set of ten and arrive at other such sets by pairing. Since some children are more proficient in counting than others, do not wait until all the paste sticks are counted. As soon as a variety of sets are shown (anywhere from one through nine sets of ten) collect the sticks that have not been counted. A chart may be made on the chalkboard with blanks in which children can record the number of sets of ten that have been counted.

_____ tens   _____ tens

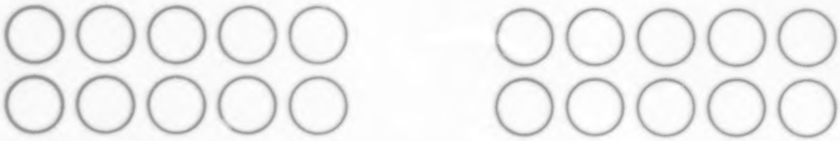
_____ tens   _____ tens

Provide enough spaces so that every child will have an opportunity to write the numeral that names the number of sets he has counted. As each child names the number of tens, the other children should count together to check his answer as he writes the numeral in the chart. This type of experience is of value since it requires the active participation of every child as well as providing practice in counting. The teacher has an opportunity to observe counting procedures and the writing of numerals. This procedure may be repeated by the class or by those students who need to attain greater proficiency in counting.

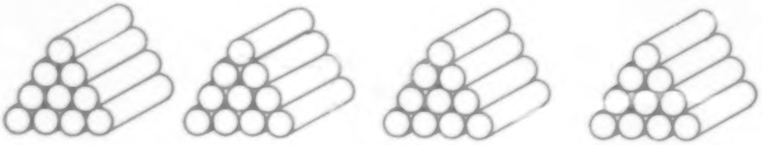
USING THE PUPIL'S BOOK, pages 112 - 113.

Pupils should work independently on these pages. Give help individually to those children who need it.

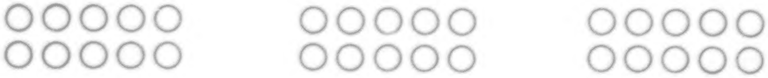
How many tens?



2 Tens

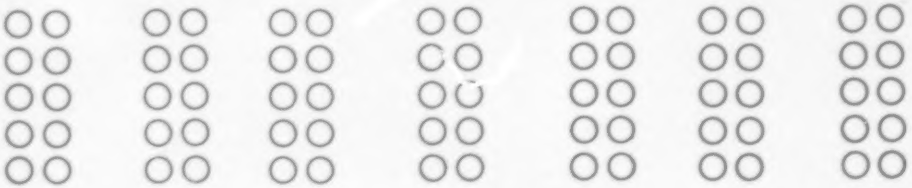


4 Tens



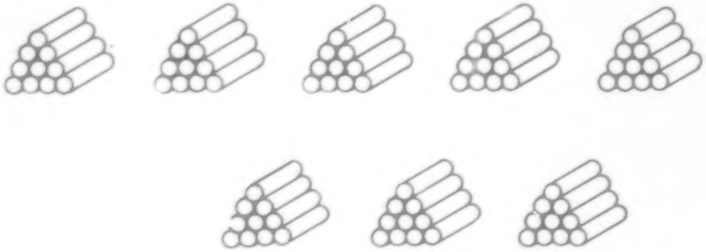
3 Tens

How many tens?



7 Tens

5 Tens



8 Tens

FURTHER ACTIVITIES:

1. How many fingers are in the set of fingers on both of your hands? Then let ten children stand in front of the class, while the other children count the ten sets of ten fingers. Have one child come to the front and count the sets of ten by starting at one end of the row of children. Have another child count by starting at the other end of the row. Did both count ten sets of ten? Did both say the counting names in the same order?

2. Let children work as partners. Have each child in turn draw around the other's outspread hands palms down. Let children color and cut out their "hands". Then paste the hands, in pairs, to a strip of wrapping paper to use in practicing counting by tens.

3. Have children count sounds as you tap the desk or a triangle. After a set of 10 taps, each child holds up one finger; after the next set of 10 taps he holds up another finger, and so on. Stop after every multiple of ten, and have children tell you how many "tens" of taps there have been. Or, each child may record the sets of ten by marking a tally mark for each of the sets.

3-2. NAMING MULTIPLES OF TEN

OBJECTIVE: To emphasize that the order of counting "tens" is the same as the order of counting "ones"; to teach the names for multiples of ten.

VOCABULARY: Row, column; ten, twenty, thirty, ..., one hundred.

MATERIALS: Spool board with 100 spools or pegboard with 100 pegs, large piece of plain tagboard or cardboard to cover rows for counting by tens; 1" x 1" cut-outs for flannel board and ten strips of ten of these made by affixing to masking tape; objects for children to use in forming sets of ten, e.g., bundles of sticks, lima beans or other small objects in plastic bags, strips of tickets, the strips of 1" x 1" cut-outs, buttons on cards.

SUGGESTED PROCEDURE:

Call attention to the rows and columns of the spool board or peg board.

How many places for spools (pegs) are there in one row? (Indicate a row.)

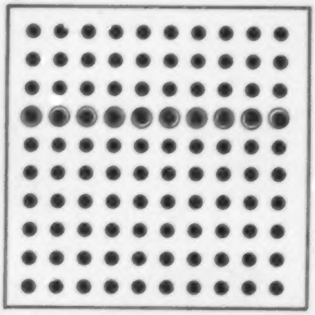
In one column? (Indicate a column.)

How many spools will each row of the board hold? (Ten.)

How many spools will each column of the board hold? (Ten.)

How do you know?

Ask a child to choose a row and fill it with spools. (If he selects a column, help him change to a row without belaboring the point. The distinction between row and column is introduced at this time. Clarification will come with use.)



How many spools are in the set on the board? (Ten.)
How do you know? (Answers will vary.)
Do we have one set of ten spools? (Yes.)

Place another row of spools on the board.

How many sets of ten spools are on the board now? (Two.)

Continue placing rows of spools on the board letting the children count them: 1 set of ten, 2 sets of ten, etc. to 10 sets of ten. Have the children count to ten so that the order will be recalled easily. Then have the rows of spools counted as you move a sheet of tagboard to expose them: 1 ten, 2 tens, 3 tens, etc.

Can we count tens in the same way we count ones? (Yes.)

Cover all but the two top rows of spools with the tagboard.

How many sets of ten spools do you see? (Two.)
Who will tell us another name for 2 tens? (Twenty.)

Continue in similar fashion, introducing any names that are not known by a child. Then let the children count the rows of spools again with you: ten, twenty, thirty, ..., one hundred.

Give each child sets of paste sticks that have been grouped into sets of ten. A rubber band may be used to hold the bundles together. After the children have counted the sets of ten, ask each child to tell how many sticks he has. The older children may count the bundles as he holds them in the air. Make sure that both names are used. For example: 2 tens, twenty; 3 tens, thirty; etc.

Follow up by asking such questions as:

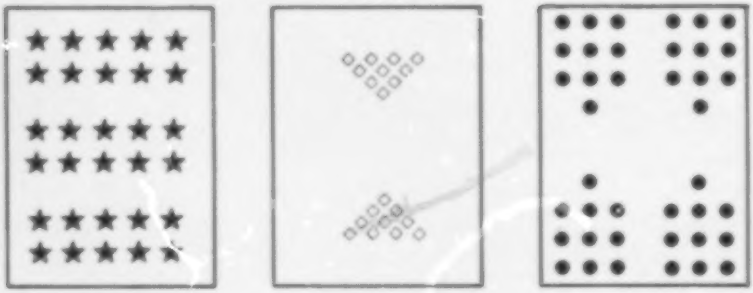
Who has 7 tens?
What is another name for 7 tens? (Seventy.)
Who has 4 tens?
What is another name for 4 tens? (Forty.)

Write numerals on the chalkboard, e.g., 9 tens, 1 ten, 5 tens. Do not write 90, 10, or 50. Ask the children who have the corresponding number of sets of 10 to display them to the class.

FURTHER ACTIVITIES

1. Provide practice in counting by tens to one hundred by asking children to count together as you display various sets of ten.
2. Use gummed stickers to make perception cards for sets of ten. Arrange the sets of ten in various patterns, but keep them

as easily distinguishable units on the cards. For example,

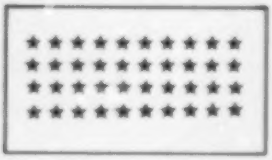


The children can be divided into two teams. One child may record the correct answers by making tallies on the chalkboard. Hold up the cards one at a time, and alternate from one team to the other as you ask the number named. The words twenty, thirty, etc. are the preferred answers. If 6 tens is given as the answer, ask for the name of the number.

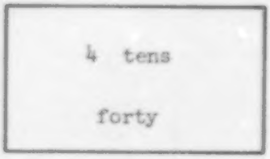
3. The teacher may serve as leader to introduce the activity; later a child may be the leader.

   This card shows forty stars.
   Forty is how many sets of ten? (Four.)

Then briefly expose the card to the children. Show the card again so that a check can be made. Following several experiences with the cards, children may use them as an independent activity. After these names have been learned, they can be written on the reverse sides. Do not, however, drill on these words or expect all children to learn them. The association of word, numeral and set representation is provided for those who may benefit.



(Front of card)



(Back of card)

3-3. APPLICATION: MONEY

OBJECTIVES: To introduce the relative value of ten cents and one dime.
To reinforce understanding of "ten" as "ten ones" or "one ten".

VOCABULARY: Dime, worth, value.

MATERIALS: 100 pennies, 10 dimes, sheet of construction paper.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Have the children gather around a table where all may see. Let them discuss briefly their experiences in using money. Show them a dime and a cent.

> If you could choose just one of these coins, which one would you choose? Why?

Note whether or not any children use the words cent, penny, dime, value or worth spontaneously. If not, introduce the words into the discussion when appropriate.

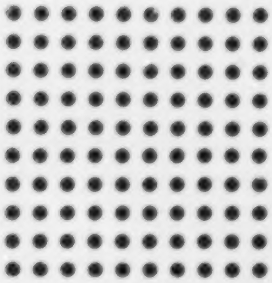
Put 100 pennies on a table and ask the children to count a set of ten pennies. Ask if anyone knows how this set of ten could be used to help in finding the number of the set of all the pennies on the table. A child may suggest arranging the rest of the pennies into sets of ten that match (are equivalent to) the set of ten that was counted.

If the suggestion does not come from a child, introduce the idea by asking the children to count to another set of ten as you arrange the second row of pennies.

> Are these two sets equivalent? (Yes.)
> Did we have to count the members of the second set? (No, just pair.)
> Watch as I place another row of pennies.
> Tell me when I have enough for another set of ten.

If there is any disagreement about the number of pennies in the third row, counting may be used as a check.

After ten sets have been arranged, hold a sheet of construction paper to cover all but the first row, and uncover rows as the children count: one ten, ten cents; two tens, twenty cents; three tens, thirty cents; and so on.



Ask questions to bring out the idea that there are ten rows of pennies with ten pennies in each row; the number of pennies in the set on the table is one hundred; there are one hundred cents altogether.

Ask how many pennies there are in the first column of pennies. Use a procedure like that used with the rows and let the children count the columns by tens.

Place a dime at the end of each row of pennies, slightly apart from the row. Note that there is a set of ten pennies for each dime, and a dime for each set of ten pennies; the number of the set of dimes on the table is ten. Use a piece of cardboard to cover the money in such a way that one row of pennies and 1 dime are exposed.

> Would you rather have ten pennies or a dime? (It doesn't make any difference.)
>
> Why? (Ten pennies have the same value as 1 dime.)

Continue the same line of questioning as you move the cardboard down the rows of pennies and dimes. Then hold up a dime and give several children a number of pennies. Ask each child if he will exchange his pennies for your dime. Regardless of his answer ask the class if they agree with him and why.

Remove the pennies from the table. Ask how much one dime is worth. Tell the children that a dime is sometimes called a "ten-cent piece"; when money is counted, dimes are counted as tens. Play "banker" and ask the children to count the dimes: "one dime, ten cents; two dimes, twenty cents; ...,ten dimes, one hundred cents". Give ten pennies or more to each of several children, letting them count ten cents and give it to the "banker" in exchange for one dime.

Does anyone know the name of a bill you get in exchange for ten dimes?

USING THE PUPILS BOOK, page 114.

Page 114 is a teaching page. Exhibit the page for all the children to see. Discuss with the children the pictures of the penny (1 cent) and the dime. Give help on reading the words "one cent", "one dime", "ten cents".

Have a child come to the front and find the picture of the kite. Have each child point to the kite in his book. Ask:

How much is the kite? (1 dime.)
How many pennies will it take to buy the kite? (10.)

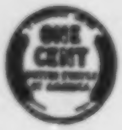
Have the child mark ten pennies with an X.

Do the same thing for the toy soldier.

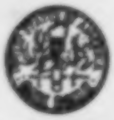
Page 115 is to be done independently. Give help where needed.

Teaching Page

Dimes and cents

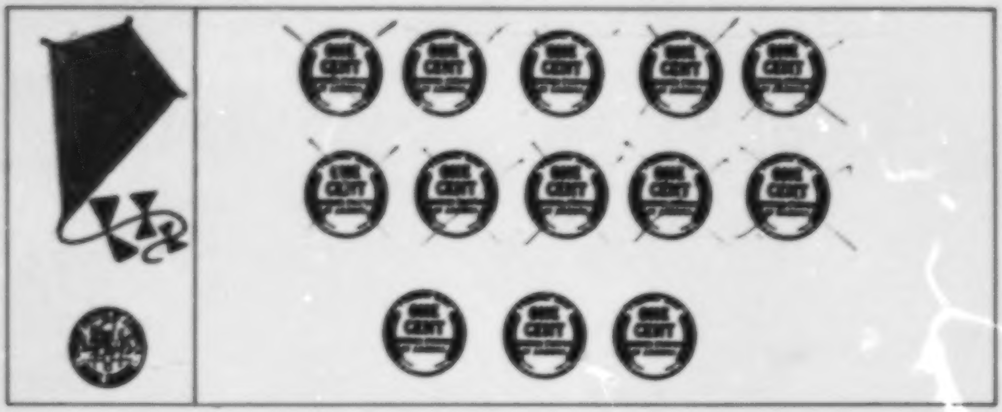


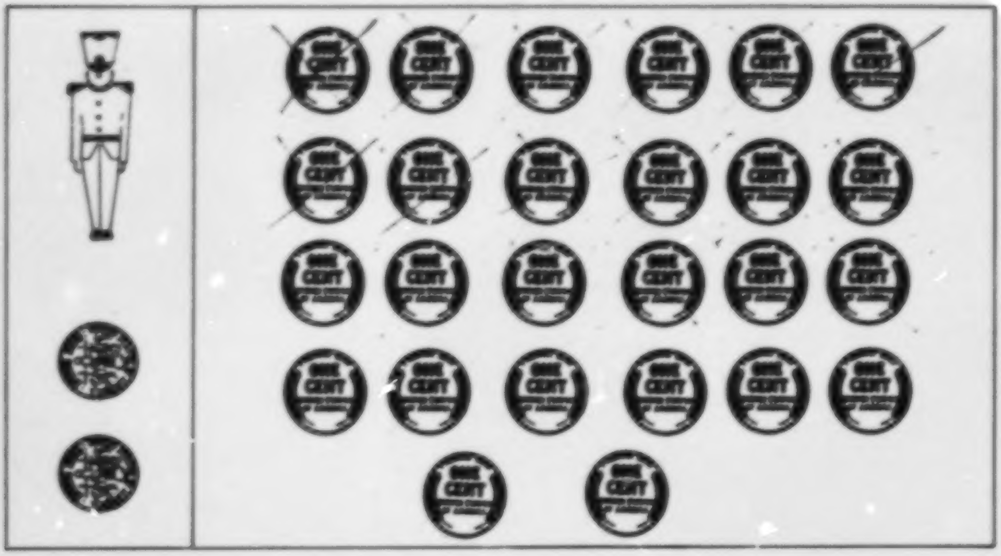
One cent



One dime
Ten cents

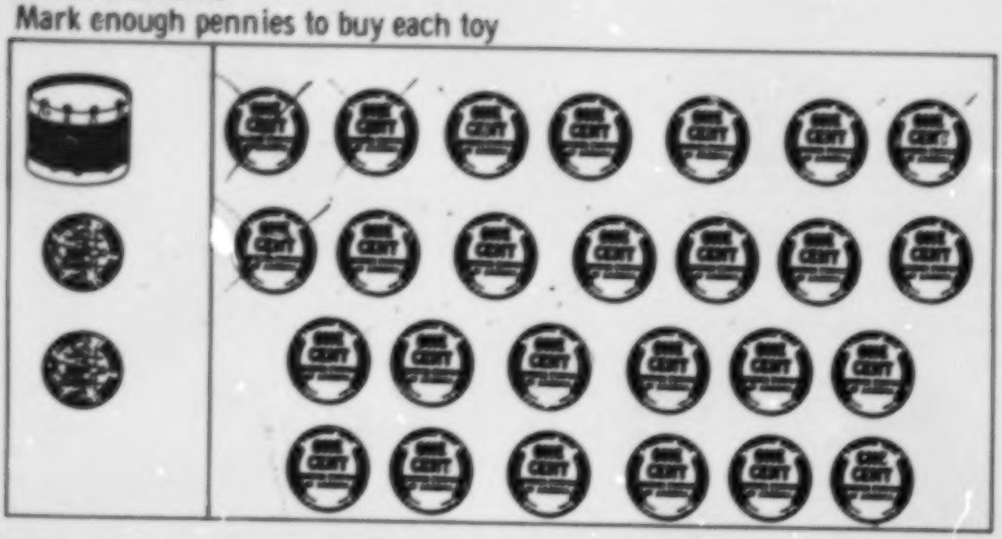
Mark enough pennies to buy each toy.

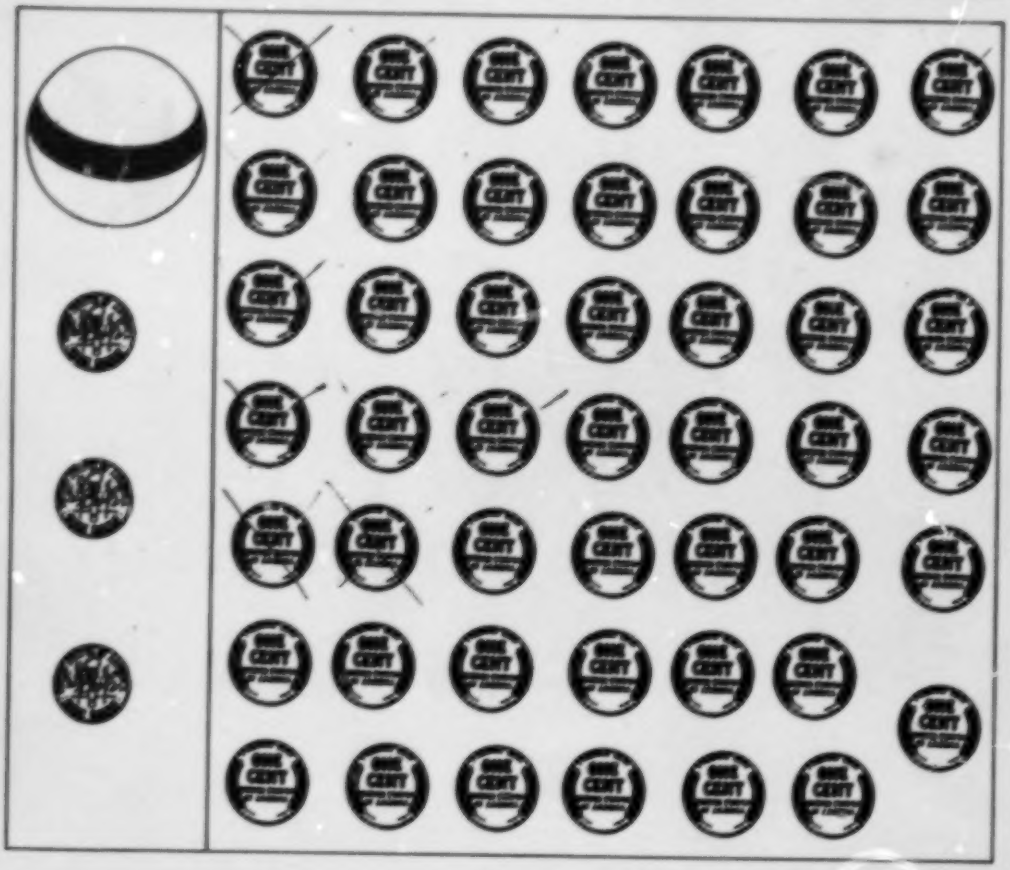




Dimes and cents

Mark enough pennies to buy each toy





FURTHER ACTIVITIES:

1. Make a chart showing the values of the coins cent, nickel and dime, and use it for discussion and review. Real coins may be fastened to tagboard with transparent tape. If you wish, some space may be left for coins of larger denomination.
2. Use opportunities that arise naturally in the classroom for counting money and discussing the values of coins.

3-4. PROBLEM SOLVING WITH SETS OF TEN

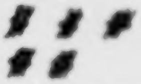
OBJECTIVE: To extend understanding of "tens" through using sets of ten in problem solving; to develop ability to solve problems involving inequality, one more ten and one less ten.

VOCABULARY: (No new words.)

MATERIALS: Sets of ten tickets, sticks, toothpicks, tongue depressors, clothes pins, plastic spoons, held together by rubber bands or tape.

SUGGESTED PROCEDURE

The intent is that problems be read or told to the children, and that no written equations be used in connection with them. The sequence is from relatively simple to more complex problems. Selections should be appropriate for the children in the class.

Give each child several sets of ten (paste sticks or slips of paper held by a rubber band). Ask the children to place a set of twenty in the middle of their desks. Then ask for a set of thirty. If the two sets are arranged in this manner  the comparison will be obvious.

Suppose these paste sticks were candies.
Which set would you choose? (The set of thirty.)
Why? (There are more.)
How many more? (Ten.)
Is 30 greater than 20? (Yes.)
How much greater? (Ten.)

If there is any uncertainty about "how many more", ask the children to show by pairing that one set of ten or ten is left over. Clear the desks and continue with the following problems.

1. Bill has twenty marbles. John has ten. Which boy has more marbles? How many more? Is 20 greater than 10? How much greater?
2. Anthony has forty sticks of gum. Joe has fifty. Which boy has more? How many more? Is 50 greater than 40? How much greater?
3. Betty has seventy pennies. Mary has sixty. Does Betty have more pennies? How many more? Which number is greater, 70 or 60? How much greater?
4. Ann has 30 blocks. Elizabeth has 40. Does Ann have more blocks? Does Elizabeth have more than Ann? How many more? Which is greater, 30 or 40? How much greater?

- Repeat the same procedure using bundles of ten. Emphasize the number that is less than another.

1. Eddie has twenty sticks of candy. Leon has ten. Does Leon have fewer pieces than Eddie? How many fewer? Is 10 less than 20? How much less?
2. Sue made 30 jumps with the jump rope. Jane jumped 20 times. Did Sue make fewer jumps than Jane? Did Jane make fewer jumps than Sue? How many fewer? Is 20 less than 30? How much less?
3. Charlie's team had 40 people up to bat during the ball game. Jimmy's team had 30. Which team had fewer chances to make a hit? How many fewer? Is 30 less than 40? How much less?
4. Kitty knows 60 people. Wanda knows 50. Does Wanda know fewer people than Kitty? How many fewer? Is 50 less than 60? How much less?

FURTHER ACTIVITIES

1. Name a number that is a multiple of ten but less than one hundred. Ask the children to use bundles of ten to represent a number greater than or less than the number named.
2. Divide the class into teams. Make it clear that only numbers named by sets of ten are to be used. Alternate in calling on members of the two teams to name a number that is either greater than or less than 20, 60, 30, etc.

Chapter 4

INTRODUCTION TO ADDITION AND SUBTRACTION

BACKGROUND

The union of disjoint sets is the basis for the concept of addition. One set joined to another is the set consisting of all objects which belong to either set. In your classroom, if the set of boys is joined to the set of girls, the result is the set of all children in your classroom. This set is called the union of the two sets joined. We may represent two sets on the flannel board, and think of joining the first set to the second, or of joining the second set to the first. Of course, the same set results so we may say that joining or union is commutative. This fact serves later to show us that addition is commutative.

The idea of joining sets underlies the arithmetical idea of addition. In a classroom, if the set of all boys is joined to the set of all girls, the result is the set of all children. The number of girls plus the number of boys is equal to the number of children. Thus 2 + 3 is by definition the number of members in the set obtained by joining a set of 3 to a set of 2. There is one complication. The sets which we join must have no members in common. We call such sets disjoint. Thus the set consisting of

John and Mary has 2 members,

the set consisting of

Mary, Sue and Jane has 3 members,

but the set obtained by joining these two sets, which consists of

John, Mary, Sue, and Jane, does not have 2 + 3 members.

The trouble is that Mary is a member of both sets, and the sets are not disjoint. The distinction between disjoint sets and sets that are not disjoint need not be pointed out to children. The use of manipulative materials and/or pictures to represent the members of sets insures that the sets will be disjoint. Consequently, the problem will not exist for children at this grade level and will not be brought to their attention until later in the curriculum.

Joining a set of 2 elements to a set of 3 elements always gives the same set as joining the set of 3 elements to the set of 2 elements. Consequently $2 + 3 = 3 + 2$. Similarly, $4 + 1 = 1 + 4$, $5 + 7 = 7 + 5$, and so on. We say that the operation of addition is *commutative*.

We wish to make very clear the sense in which *equality* and the equals sign, (=), are used in mathematics. We write, for example, $3 + 4 = 7$, or $7 = \text{VII}$, because "3 + 4", "7" and "VII" are all names for the same thing, the number seven. Any statement of equality means that the symbols to the left of the equals symbol and the symbols to the right of the equals symbol are names for the same thing. A number, like a person, may have many names. All of the following

$$7,\ 3 + 4,\ 5 + 2,\ 8 - 1,\ 49 \div 7,\ \frac{14}{2},\ \text{VII}$$

are names for the number seven. The reason we adopt this meaning for equality is that we *always* want to be able to substitute equals for equals. Thus, for example, if we know that $6 = 3 + 3$, we can infer that the sum of 7 and 6 is the sum of 7 and $3 + 3$.

Addition, which we denote +, is an *operation*. If we apply the operation + to a pair of numbers, say 3 and 4, the result is another number, $3 + 4$ or 7. We write $3 + 4 = 7$ because "3 + 4" and "7" are names for the same number. It is also true that $3 + 4 = 6 + 1$, and indeed, if we ask a child, "What is $3 + 4$ equal to?" then all of the following are correct answers: 7, $6 + 1$, $3 + 4$, $4 + 3$. The procedure which we call addition really amounts to finding the common name, a name not involving +. For example, when we ask that 3 and 4 be added, we want the common name 7, not the name $3 + 4$ or $2 + 5$, etc.

We say that one set is a *subset* of another if each member of the first is a member of the second. The set of girls in a classroom is a subset of children, and the set of all tricycles is a subset of the set of all toys. The set consisting of John and Mary is not a subset of the set of all girls because John does not belong to the set of girls.

Let us list all possible subsets of the set consisting of an apple and an orange. It is clear that there are two subsets which have just one member each: the set consisting of the apple and the set consisting of the orange. Are there other subsets? What of the empty set? Is it true that every member of the empty set is a member of the set consisting of an apple and an orange? If not, then some member of the empty set must fail to belong to the apple-and-orange set, and this is impossible since the empty set has no member. We must therefore agree that the empty set is a subset of the set consisting of an apple and an orange, and in fact that the empty set is a subset of every set. (Later we will relate this to statements like $5 - 0 = 5$.) Finally, we may ask if this apple-and-orange set is a subset of itself. Is it true that every member of the set consisting of an apple and an orange belongs to the set consisting of an apple and an orange? This is obviously true, so we agree that the apple-and-orange set is a subset of itself. In fact, every set is a subset of itself. (We will relate this fact to statements like $7 - 7 = 0$.)

Suppose we are given a set and a subset of it and we *remove* the subset. The *remaining set* consists of those objects which belong to the original set but not to the subset. If we remove the set of boys from the set of children in the classroom, the remaining set is the set of girls.

We describe *subtraction* in terms of removing a subset of a set: $5 - 3$ is the number of members in the remaining set if a set of 3 elements is removed from a set of 5 elements. Thus, if John has 5 marbles and his older brother takes 3 marbles, then the remaining set has $5 - 3$ members.

Note that the operation of subtraction does *not* possess the commutative property. That is, $a - b \neq b - a$. Since the child at this time is restricted to the set of whole numbers, an equation such as $3 - 5 = \square$ cannot be completed. Subtraction is defined only if b is less than or equal to a.

Despite this lack of commutativity there is, however, a *close relation between addition and subtraction*. If we join a set to another set and then remove it, the remaining set is just the original or "starting" set. If we remove a subset from a set and then join it to the remaining set, then we again have the original set. We sometimes say that joining a set, and removing the same set, are *inverse* operations, in the sense that doing these in succession to any set always gives back the original set. This fact about manipulation of sets shows us something about addition and subtraction. If we add 2 to a number and then subtract 2, we have the original number. If we subtract 2 and then add 2, we again have the original number. Thus, adding 2 and subtracting 2 are *inverse operations*, in that doing these in succession to any number always gives the original number. Of course, adding 4 and subtracting 4 are also inverse operations, and so on.

There is one matter of notation which need to be made clear. In some of the equations in this chapter we have left boxes for the children to write in (for example, $2 + \square = 5$). These boxes are nothing more than places for the children to write; they are not interpreted as placeholders or variables, as is the case in some of the other mathematics programs. We introduce variable notation in the second grade, where we write, for example $2 + n = 5$.

4-1. Joining

OBJECTIVE: To introduce the operation of joining and its commutative property.

VOCABULARY: Join.

BACKGROUND NOTE:

If one set is joined to another, the result is the set whose members are those things which belong to either of the sets. This is preparation for the concept of addition.

MATERIALS: Materials for flannel or magnetic board demonstration; beans bottle caps, buttons, blocks, and spools.

SUGGESTED PROCEDURE:

After the children are familiar with the ideas of set and member, they should be ready to understand the concept of join. Some will understand the word as they have heard it used in other situations.

You might start with a demonstration at your desk, at a low table, or on a flannel or magnetic board where all can see. On one side of the table, place a set of ten or fifteen various sized buttons and on the other side a set of blocks, another kind of material, or another set of buttons which can be distinguished from the first set. Each set should contain too many members to be counted quickly as no counting is wanted at this time.

After the sets have been described by the children, proceed somewhat as follows:

> We will move this set of buttons over to JOIN it to the set of blocks.
> When we join these sets, we have a new set.
> What are the members of this set? (Buttons and blocks.)

Touch a member of the set formed by joining the set of buttons to the set of blocks.

> Is this button a member of the set of buttons? (Yes.)
> Is this button a member of the set of blocks? (No.)
> Is it a member of the set of buttons and blocks? (Yes.)
> Is each member of the set of buttons a member of the set of buttons and blocks? (Yes.)

Is each member of the set of buttons and blocks
a member of the set of buttons? (No.)

Discuss several other members of the set in a similar manner. Through this discussion the children should be helped to understand that each member of the new set (buttons and blocks) was a member of either the set of buttons or the set of blocks and that each member of the set of buttons and the set of blocks is a member of the new set.

Move the set of buttons back to its original position.

Now let's move the set of blocks over to join
the set of buttons.
When we join the set of blocks to the set of
buttons, we have a new set.
What are the members of this set? (Buttons and blocks.)
How is this set like the set we had when we joined
the set of buttons to the set of blocks? (It has
the same members.)

You should follow the same procedures using a variety of materials. Emphasize the ideas that one set is joined to the other to form a new set; each member of the new set is a member of one or both of the sets joined; the order of joining the sets does not change the new set.

We have our set of books about animals here at the
library table.
On my desk is the set of new animal books that I got at the
library today.
John, will you bring the set of new books and join it to
the set here at the table?
How would the new set be different if we took the set of
animal books from the library table to join the set of
books on my desk? (No difference.)

You may prefer to use the following activity the day after introducing the concept of joining. See that each child has a set of spools and a set of beans or other sets of two different kinds of material on his desk, one set on each side. Again have too many members in each set to be counted quickly.

Put your right hand on the set of beans on your desk. Now move this set over to join the set of spools.
What is the new set on your desk? (A set of beans and spools.)

Ask the children to pick up a member of the new set which was not a member of the set of beans. (A spool.) Pick up a member of the new set which was not a member of the set of spools. (A bean.) Ask the children to pick up a member of the new set that was not a member of the set of beans or the set of spools. (None.)

Move the set of beans and spools back where they were when the lesson started. This time, move the set of beans over to join the set of spools.

What set do we have when we join a set of beans to a set of spools?
(A set of beans and spools.)
How is the set like the set we had when we joined the set of spools to the set of beans. (The members of the new set are the same as before.)

Emphasis is made here on the idea that the order of joining sets does not affect the new set.

FURTHER ACTIVITIES

Use sets of children to illustrate that the order in which the sets are joined does not affect the membership of the new set which is formed.

4-2. JOINING SETS AND COUNTING

OBJECTIVE: To prepare for the operation of addition by introducing the joining of sets in association with the spoken names of numbers 0 through 9.

VOCABULARY: (Review.) Joining, member, set, empty set, set with one member.

SYMBOLS: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

MATERIALS: Materials for flannel board, sets of small objects; perception cards (see instructions under Further Activities), a box of small objects for each child.

SUGGESTED PROCEDURE:

As a pre-addition activity, you will find it useful to establish an understanding of joining sets and counting, counting both the sets with which they started, and the resulting set. An introduction by means of a flannel board demonstration follows. (The same procedure, with slight modification, is applicable to chalkboard illustrations.)

Place a set of objects on the flannel board and have the set described. (Example: set of apples, set of red apples, set of 5 apples.)

Identify the number of the set and place the set to one side of the board. On the other side of the board, place another set. (Example: set of 3 oranges.) Identify the number of this set as you did the first.

We now have a set of 5 apples on our board and a set of 3 oranges.
Let's put the oranges with the apples.
What have we done to the two sets? (Joined the set of oranges to the set of apples.)
We began with a set of 5 apples and a set of 3 oranges.
How many members do we have in the new set that we just made by joining the two sets? (8.)

By moving the set on the flannel board, the children are able to see the joining of the separate sets and formation of the new set.

After the children are familiar with the verbal presentation, cards with numerals written on them, one numeral per card, should be placed on the flannel board or table during the discussion and the appropriate card displayed after the number of members of a given set is identified.

What is the number of members in the set of apples? (5.)
What is the number of members of the set of oranges? (3.)
What is the number of members in the set of apples and oranges? (8.)

Place the oranges on the flannel board. Then join the set of apples to the set of oranges and ask that this set be described.

When we join the set of apples to the set of oranges, what is the new set? (A set of oranges and apples.)
What is the number of members of the set of apples? (5.)
Display the numeral card with 5 written on it.
What is the number of members of the set of oranges? (3.)
Display the numeral card with 3 written on it.
What is the number of members of the new set? (8.)
Display the card with 8 on it.
How is the set we formed by joining the set of apples to the set of oranges like the set formed by joining the set of oranges to the set of apples? (They have the same number of members. They have the same members. We changed the order in which we joined the sets but this does not change the set which is formed.)

- Place a set of 4 rabbits on the flannel board. Ask a child to name the number of members in the set. Place a card with this numeral on the flannel board. Place a set of 3 kittens on the flannel board. Ask a child to name the number of members in the set of kittens. Place a card with this numeral on the flannel board. Ask a child to join the set of kittens to the set of rabbits.

What are the members of the new set? (Kittens and rabbits.)
What is the number of members in the new set?
Place a card with this numeral on it on the flannel board.

A discussion such as the following may serve to evaluate how well children remember which set had a given number of members.

Remove the set of kittens and rabbits from the flannel board and point to the card with the numeral 3 on it.

What set had this number of members? (The set of kittens.)

Point to the card with the numeral 4 on it.

What set had this number of members? (The set of rabbits.)

Point to the card with the numeral 7 on it.

What set had this number of members? (The set of rabbits and kittens.)

If we had joined the set of rabbits to the set of kittens would we have the same number of members in the set of rabbits and kittens as in the set we had earlier? (Yes.)

FURTHER ACTIVITIES:

1. Perception cards made by the teacher help to visualize the joining of sets. Several cards should be made for each number so that children do not merely learn to recognize a pattern. Magazine pictures pasted on heavy paper are also useful. Some advertisements have excellent pictures and are fun for children to find. They also enjoy making these cards themselves. Hold up one card and have the children identify the number of members. (Identification may be made either by recognition or counting, or a combination of both.) Place this card on a stand and hold up another card using the same procedure. Join the two sets and identify the number of the resulting set.

2. Sets of objects such as books, writing equipment, art suppiies, and blocks may be placed on a demonstration table. First, identify two sets and join one to the other to form a new set. Remember to identify the number of members in each set as well as the number of members in the new set.

   Manipulative materials may be used at the pupils' desks to be joined to form new sets. Proceed as before, being certain to use orally the number of both of the set with which you started and the resulting set.

3. Use sets of class members like ball monitors, errand helpers, etc. Join one set to another and name the number of the resulting set. (Example: ball monitors 2, errand helper 1, resulting set 3.)

4. An overhead projector, if available, is useful in developing this lesson. It provides an excellent means of viewing the sets, writing names of numbers associated with these sets, and showing the result of joining one set to the other.

4-3. JOINING SETS AND ADDING NUMBERS

OBJECTIVES: To develop an understanding of addition.
To use the terms plus and equals and the symbols + and = to write equations.

VOCABULARY: Add, plus, equals, number, equation.

SYMBOLS: +, =

MATERIALS: Set of books, set of blocks, large sheets of paper divided into three columns for a chart.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Ask a child to pick up a set of 2 books from the shelf. Have the child place the books on the table, saying that this is our first set. Explain to children that we will keep a record of our work and record this number in the first column of the chart.

2		

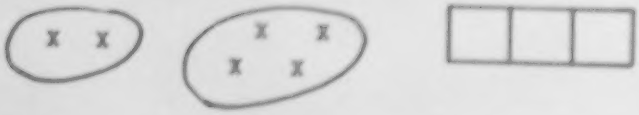
Ask a child to pick up a set of 3 books. We want to join this set to the first set by putting these books with the books on the table. You should then explain that you will record this number of books (set being joined) in the second column of the chart.

As the chart is developed, continue to point out to the children that a numeral in the left hand column shows the number of objects in the set we started with and a numeral on the middle column shows the number of objects in the set joined to the first set. Join the set of 3 books to the set of 2 books.

How many members are in the new set of books? (5.)

Record 5 in the right hand column of the chart. Explain to the children that this numeral (5) names the number of members in the new set when the second set is joined to the first set. Continue with other examples, recording the numbers for each example on the chart.

Make a drawing on the chalkboard like the one given below.



What is the number of the first set? (2.)
Where should I write the numeral that names this number? (In the first box or column.)
How many members in the second set? (4.)
Where should I write the numeral that names this number? (In the second box or column.)
If I join these two sets, (draw a ring around the sets to indicate the union) how many members would there be in the new set? (6.)
Where should I write the numeral that names this number? (In the last box or column.)

You may wish to repeat this procedure using other set representations.

USING THE PUPIL'S BOOK, pages 116-117.

Page 116 is a teaching page. Have one child come to the front of the room where page 116 is displayed. Point to the set of trees on the left. Ask

How many members are in this set? (2.)
Where are you going to write the numeral that names this number? (In the first box.)
Please trace over the dotted numeral in the first box in your book.
How many members are in the second set? (3.)
Where are you going to write the numeral that names this number? (In the second box.)

Please trace over the dotted numeral in the second box.
If we join these two sets, how many members will there be in the new set? (5.)

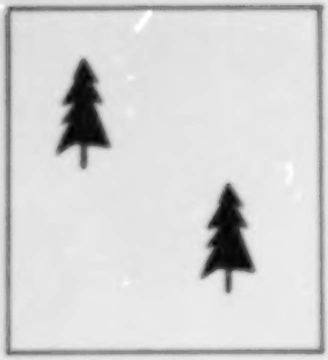
Where are you going to write the numeral that names this number? (In the third box.)
Please trace over the dotted numeral in the last box.

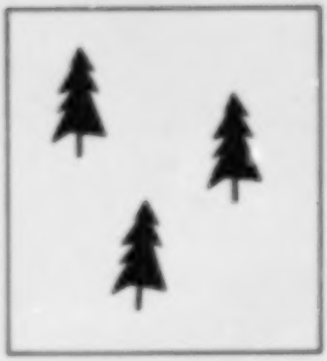
Continue with the other two sets. Be sure the children record the information as it is given.

Use page 117 for independent work.

Teaching Page

How many?





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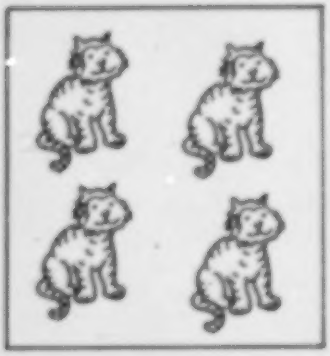
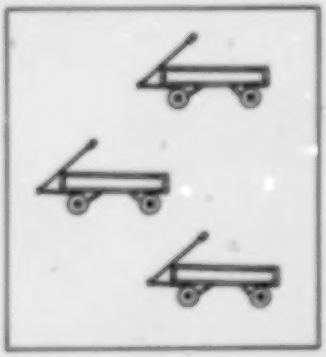
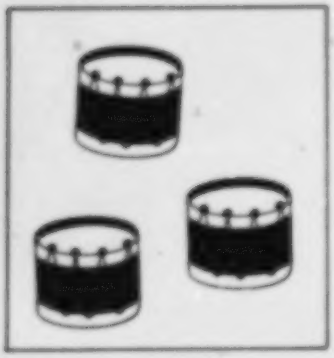
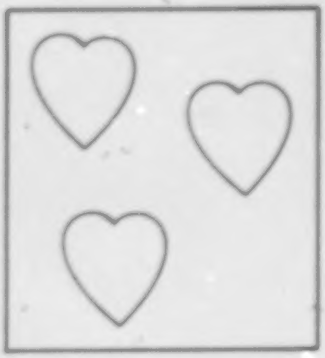
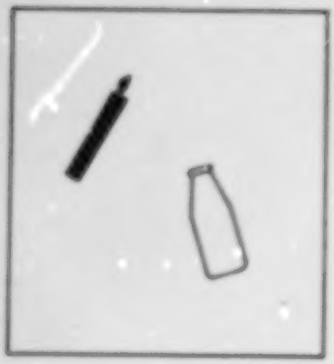
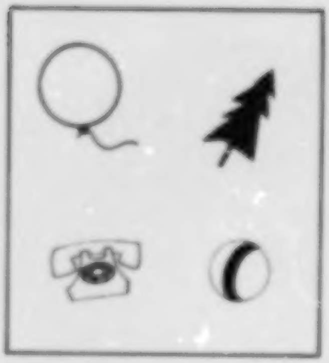




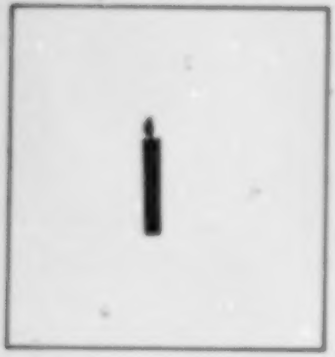
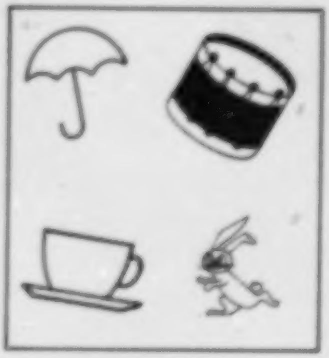




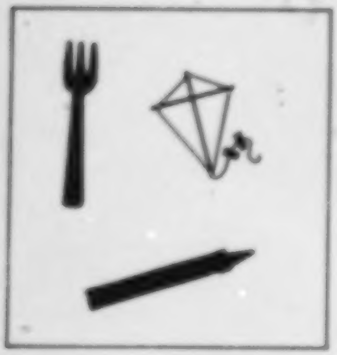
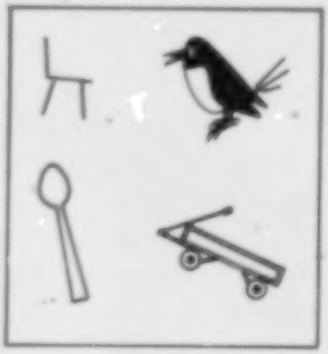
How many?

2	4	6

1	4	5

[illegible]	[illegible]	[illegible]

- Continue with the information on the chart.

Look at the record of our work with the sets of books.
We can use what is written to make an equation.
In the equation we say, "two plus three equals five".
To the number 2 we are ADDING the number 3.
The result is the number 5.

Extend the chart and write the equation, 2 + 3 = 5 on the chart.

This is how we write the equation.

Continue to extend other rows of the chart in a similar manner.

a	2	3	5	2 + 3 = 5
b	1	2	3	1 + 2 = 3
c	2	0	2	2 + 0 = 2
d	1	1	2	1 + 1 = 2
e	0	4	4	0 + 4 = 4
f	2	1	3	2 + 1 = 3
g	4	0	4	4 + 0 = 4

Sometimes we need the same numbers, but they did not come in the same order (b and f, e and g).
Sometimes the number for the new set was the same as the number of members in the one of the sets (c, e, g).
Sometimes the number for the new set was greater than either of the other numbers (a, b, d, f).
The number for the new set was never less than either of the other numbers.

SOLVING PROBLEMS

Draw several of these forms (____ + ____ = ____) on the chalkboard before beginning the lesson. Provide each child with a set of blocks or other set objects. As each statement of the problem is given, the children should represent the set. Ask for the number of the set and where this numeral should be written in the forms provided for the equations. If the children answer in terms of the first, second, or third space or mark, ask if they mean before or after the plus sign or before or after the equal sign. This will encourage the use of this language and help in the identification of these symbols.

1. Anthony had two balloons.
   His father gave him two more.
   How many balloons does he have now?

2. Ann ate three sandwiches for lunch.
   Then she ate 2 cookies.
   How many things did she eat?

3. Betty bought 4 new dresses one day.
   She bought 2 more the next day.
   How many dresses did she buy altogether?

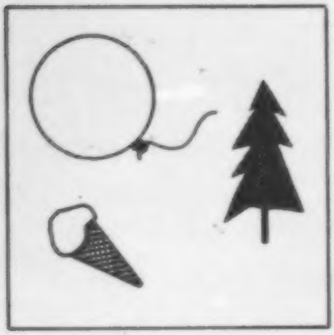
4. Johnny went downtown. He bought 3 candy bars.
   Then he went into another store and bought 3 more.
   How many candy bars did he buy?

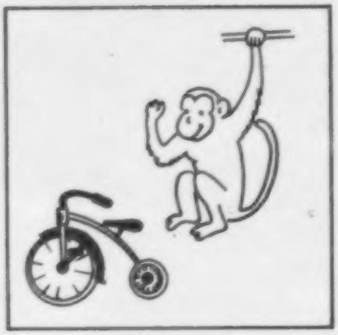
Part of the mathematics time on several successive days should be devoted to solving similar problems. Various children may be asked to write the numerals for the equation as the other children "show" the set objects on their desk. In this way, the teacher will have an opportunity to observe how well individual children have learned to make the numerals. If children can complete equations on the chalkboard, they will be in a better position to complete the work in the textbook successfully.

USING THE PUPIL'S BOOK, pages 118-119.
Work with the children to complete page 118. Be sure that the numeral that names the number of each set is recorded in the proper space before considering the union of the sets. Those children who are successful may be permitted to use page 119 for independent work.

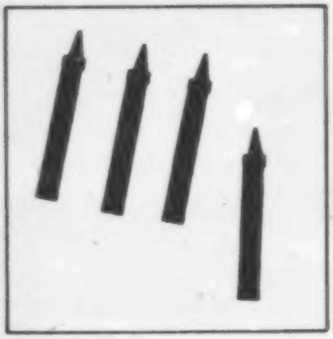
Teaching Page

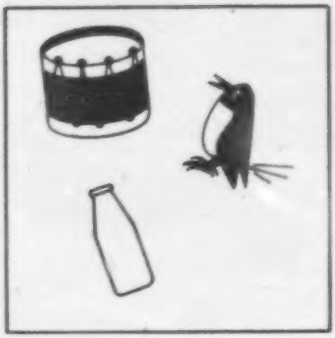
How many? Write the equation





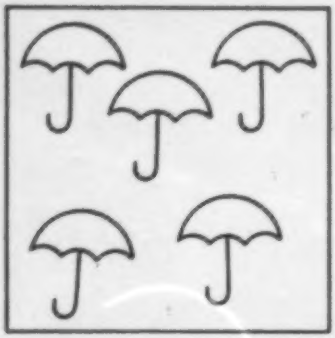
______ + ______ = ______

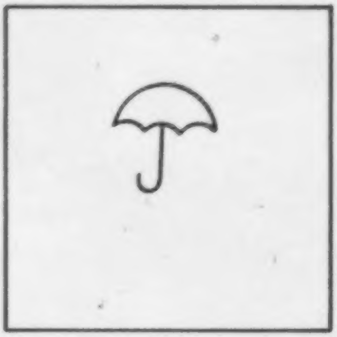




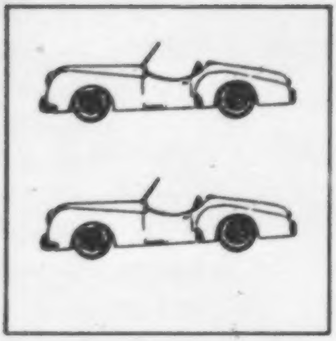
______ + ______ = ______

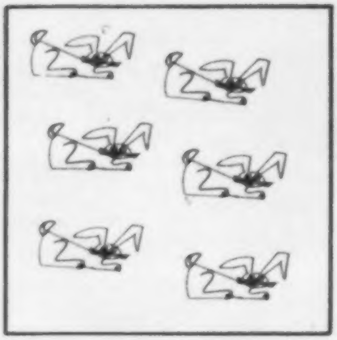
How many? Write the equation





5 + 1 = 6





2 + 6 = 8

- Provide each child with a set of blocks or other set materials. Write several equations, such as 3 + 1 = ☐ , 2 + 2 = ☐ , 5 + 0 = ☐ , etc. on the chalkboard. Help the children use materials as an aid in completing these equation. Working with concrete materials is essential as it provides every child with a method for finding the sum. Encourage their use.

USING THE PUPIL'S BOOK, page 120.

Provide set objects which the children can use to find the sum.

Pages 121-125.

Use these pages for independent work but provide every child with a set of objects.

The equations on page 121 have been arranged in this particular sequence in the hope that children who have not developed the concept that a whole number plus one is equal to the next whole number will do so.

The sequence on pages 122 and 123 is also deliberate. Hopefully, some children will observe the pattern.

The equations on pages 124 - 125 should help children generalize the commutative property of addition. Commenting that the order in which two numbers are added will not affect the sum is effective with some children. Others will have to observe this for themselves.

The experiences provided in this section are of an exploratory nature. It is not expected that children will necessarily master the basic addition facts that have been introduced. However, there is a big gap between the use of concrete materials and the abstract work on the text page. The use of the text pages provided in this section are suggestive of the type of exercises which may bridge this gap. Children will need a great deal of practice in using concrete materials to aid them in completing equations. You undoubtedly will want to provide other worksheets suitable for the needs of your class or for individual members of the class.

Teaching Page

Equations

$4 + 1 = \boxed{5}$

$3 + 0 = \boxed{3}$

$2 + 2 = \boxed{4}$

$1 + 5 = \boxed{6}$

$2 + 3 = \boxed{5}$

$0 + 6 = \boxed{6}$

$1 + 3 = \boxed{4}$

$2 + 4 = \boxed{6}$

Equations

$0 + 1 = \boxed{1}$

$1 + 1 = \boxed{2}$

$2 + 1 = \boxed{3}$

$3 + 1 = \boxed{4}$

$4 + 1 = \boxed{5}$

$5 + 1 = \boxed{6}$

$6 + 1 = \boxed{7}$

$7 + 1 = \boxed{8}$

$8 + 1 = \boxed{9}$

Equations

$2 + 0 = \boxed{2}$

$2 + 1 = \boxed{3}$

$2 + 2 = \boxed{4}$

$2 + 3 = \boxed{5}$

$2 + 4 = \boxed{6}$

$2 + 5 = \boxed{7}$

$2 + 6 = \boxed{8}$

$2 + 7 = \boxed{9}$

Equations

$$3 + 0 = \boxed{3}$$

$$3 + 1 = \boxed{4}$$

$$3 + 2 = \boxed{5}$$

$$3 + 3 = \boxed{6}$$

$$3 + 4 = \boxed{7}$$

$$3 + 5 = \boxed{8}$$

$$3 + 6 = \boxed{9}$$

Equations

$2 + 1 = \boxed{3}$

$3 + 1 = \boxed{4}$

$2 + 3 = \boxed{5}$

$4 + 1 = \boxed{5}$

$2 + 4 = \boxed{6}$

$1 + 2 = \boxed{3}$

$1 + 3 = \boxed{4}$

$3 + 2 = \boxed{5}$

$1 + 4 = \boxed{5}$

$4 + 2 = \boxed{6}$

Equations

$4 + 3 = \boxed{7}$

$2 + 5 = \boxed{7}$

$5 + 0 = \boxed{5}$

$5 + 1 = \boxed{6}$

$2 + 6 = \boxed{8}$

$3 + 4 = \boxed{7}$

$5 + 2 = \boxed{7}$

$0 + 5 = \boxed{5}$

$1 + 5 = \boxed{6}$

$6 + 2 = \boxed{8}$

4-4. SUBSETS

OBJECTIVE: To introduce the idea of subset of a set.

VOCABULARY: Subset; (review) set, collection, member.

BACKGROUND NOTE:

One set is a subset of a second set if each member of the first set is a member of the second. Thus the set of all dogs is a subset of the set of all animals because each dog is an animal, but the set of all animals is not a subset of the set of all dogs because there are animals which aren't dogs. This lesson is preparation for the concept of subtraction, which we will describe in terms of removing a subset of a set and identifying the number of members in the remaining set.

MATERIALS: Sets of objects, materials for flannel board displays, colored construction paper (several sheets of different colors), jars of multi-colored beads, rhythm instruments, sets of play dishes.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

The concept of a subset of a set may be developed as follows:

Will each member of the set of children in our room raise one hand?
Do all the boys have one hand up? (Yes.)
Do all the girls have one hand up? (Yes.)
Everyone put his hand down.

Now, let's have the girls stand.
Is Jane a member of the set of girls? (Yes.)

Repeat this question about several girls; then ask the question inserting a boy's name.

Is Michael a member of the set of girls? (No.)
Michael is NOT A MEMBER of the set of girls.
Of what set is he a member? (The boys.)
Will the boys stand?

Repeat as with the girls.

We have seen that there is a set of children in our room.
We also know that there is a set of boys and a set of girls.

We say that the set of girls is a SUBSET of the children because every member of the set of girls is a member of the set of children.
The set of boys is a subset of the set of children because every member of the set of boys is a member of the set of children.

Ask the boys to stand again. Then ask three children to raise their right hands.

Tom, Billy, and Johnny have each raised their right hands.
Are these three boys members of our set of boys? (Yes.)
However, they have their hands raised.
We call the set of boys in our class with their hands raised a subset of the set of boys in our class.

Repeat the above process with the girls: set of girls and set of girls holding books; or, the set of girls and girls wearing white shoes. Do not stop at only one such activity (for example, raising the right hand), leaving the impression that only by raising the right hand is a subset described.

This exercise can be developed further using the entire set of children and describing subsets of this set.

- At this point, you may wish to use a chalkboard or flannel board to develop further the idea of subset of a set.

Place a set of "fruit" on the flannel board. Talk about the set of apples as being a subset of fruit, the set of pears as a subset, etc.

It is important to have experiences naming subsets whose members are not selected on the basis of size, color, or use, or children develop the misconception that a subset is a subset because the members belong together for some of these reasons.

CAUTION:

It is necessary to identify the set first before talking about a subset *of* that given set. We do not define a *subset* without a reference set.

- Place a set of materials on the flannel board.

How can we describe this set? (A set with seven members. A set of flannel objects. A set of a tree, a star, a cup, a cone, a kite, a moon, and a ball.)
Is the set of a star and a cup a subset of the set of flannel board objects? (Yes.)

Is the set, a ball, a subset of the set of flannel board objects? (Yes.)
Is the set of a swing and a seesaw a subset of the set described? (No.)

Emphasize that each member of the subset must be a member of the given set.

Is the set of a ball, a tree and a watermelon a subset of the set of flannel board materials? (No.)

Then proceed with these questions.

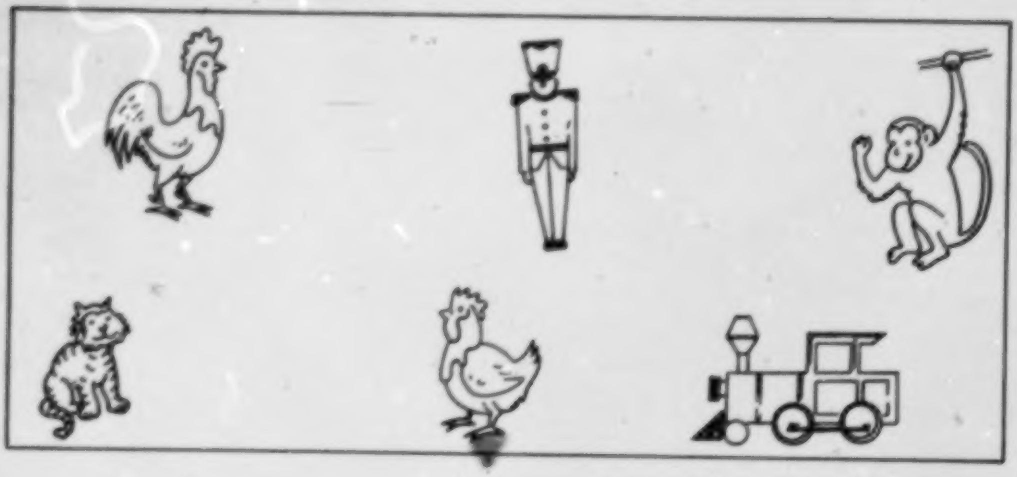
Is there any member of the set "cup, cone, kite" that is not a member of the set of flannel board objects? (No.)
Is the set "cup, cone, kite" a subset of the set of flannel board objects? (Yes.)
Yes, the set "cup, cone, kite is a subset of the set of flannel board objects.
Is there any member of the empty set that is not a member of the set of flannel board objects? (No.)
Is the empty set a subset of the set of flannel board objects? (Yes.)

USING THE PUPIL'S BOOK, pages 126-127.

These pages are class exercises. After identifying the members of the set, make a ring around the letter which names a subset of the set in the frame at the top of the page.

Teaching Page

A set and some of its subsets



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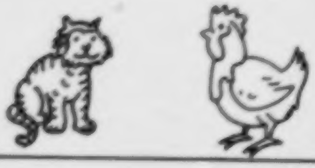

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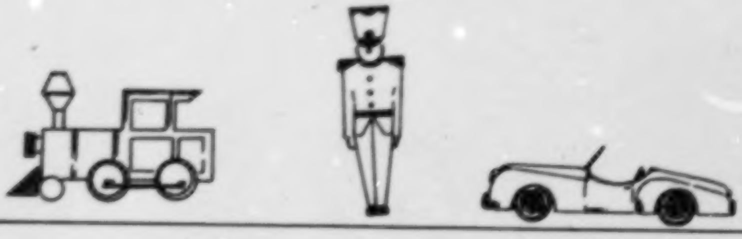

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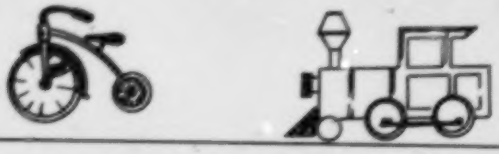

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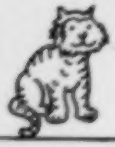
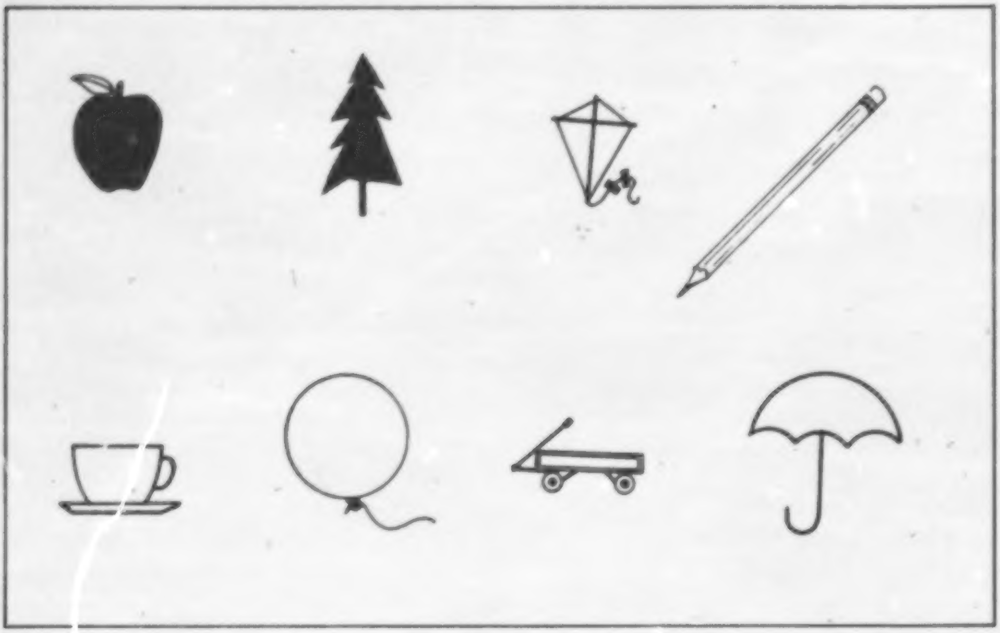


A set and some of its subsets



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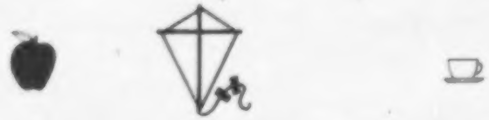

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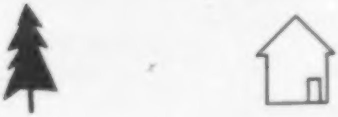
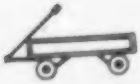


[✓ C.]



[✓ D.]

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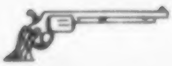

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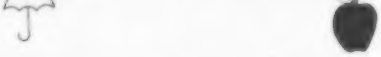


Exhibit a set of objects on the flannel board. Identify the number of members of the set. Place this numeral on the flannel board. Place a piece of yarn around a subset of the set of flannel objects. Ask the children to name the number of members in the subset. Place the numeral for this number on the flannel board. Be certain to indicate that the set is a subset of itself by placing the yarn around all the members of the set and asking for the numeral that names the number of this subset. Similarly, indicate that the empty set is a subset of every set by placing the yarn in a ring on the flannel board without any members within the ring. Ask for the numeral that names the number of members in this subset. Place this numeral on the flannel board.

After this experience, children should be ready for the following independent work:

- Distribute sets of small objects, nine or less per child. Give each child a piece of yarn. Ask the children to use the yarn to make a ring around a subset of the set of materials on the desk. Give each child a half sheet of paper which has been labeled:

Set	Subset

Tell the children to record the number of members in the set on his desk under set, make the ring to show a subset of the set and then record the number or members in the subset. Repeat activity, asking children to show other subsets of this set.

4-5. REMOVING SETS AND THE REMAINING SET

OBJECTIVE: To introduce the ideas of removing sets, the remaining set and the use of these ideas.

VOCABULARY: Remove, remaining set.

BACKGROUND NOTE:

This is further preparation for the concept of subtraction.

MATERIALS: Materials for flannel board such as apples, bananas, oranges, pineapples, cherries, small objects such as sticks and buttons.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

On a table, place a set of beans with more members than can easily be counted. Tell a child to remove a subset of the set. When we remove a set, the set that is left is the remaining set.

Pointing to the remaining set.

These observations should be made in relation to the removing of a set and the remaining set:

> Each member of the set removed was a member of the set with which we started.
>
> Each member of the remaining set is a member of the set with which we started.
>
> Each member of the starting set is a member of either the set removed or the remaining set.

● On the flannel board, place a set of fruit containing six or severn members. Ask a child to describe the set. It may be a set of fruit, such as apples, oranges, pineapples, cherries, bananas.

> Let's pretend that we are going on a picnic today.
> We will each take a sandwich for ourselves and something to share with the whole group.
> On the flannel board is the set of fruit which Dick brought.
> How many members are in the set? (7.)
> Jerry wanted some of the fruit to eat with his sandwich.
> Jerry will take some of the fruit from the set.

Ask a child to take some fruit from the set displayed.

This is a set of fruit Jerry wanted to eat.
Is it a subset of the fruit? (Yes.)
When Jerry took the subset of fruit from the set of fruit, he removed it from the set.
What is the number of members in the set removed? (2.)

Ask a child to describe the set remaining on the flannel board. (Cherry, orange, apple, pineapple, and banana.) The remaining set is the set that is left after we have removed a subset from the starting set.

Is the remaining set a subset of the starting set? (Yes, because all members of the remaining set must have been members of the starting set.)
How many members are in the remaining set? (5.)

Call attention to the many situations in the school day experience which could illustrate a set removed and the remaining set, where the starting set is the children in your classroom; for example, the set of children who went home for lunch today (the set removed) and the set of children who ate lunch at school today (the set remaining).

4-6. REMOVING SETS AND SUBTRACTING NUMBERS

OBJECTIVES: To review the idea of removing a subset.
To use number in relation to this set experience.

VOCABULARY: Minus, subtract.

SYMBOLS: -

BACKGROUND NOTE:

Subtraction is described in terms of removing a subset: 7 - 3 is the number of members in the remaining set if a subset of 3 members is removed from a set of 7 members. We shall later consider other descriptions (which include the missing addend description).

MATERIALS: Set of flannel cutouts, numeral cards, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, chart to display numeral cards, and set materials for the children.

SUGGESTED PROCEDURE:

PRE-BOOK ACTIVITIES:

Form a set with 5 objects on the flannel board. Display the set of numerals and ask if a child can find the card for the number of members in the set. Place the card on the left at the bottom of the flannel board. Ask a child to describe a subset of the set. Remove the subset and place it on the flannel board apart from the set with which you started.

How many members are in the set that was removed? (2.)

Ask a child to find the numeral card which tells this number and place it on the flannel board at the right of the first card.

What do we call the set that is here (indicate the part that was not moved)? (The remaining set.)
How many members are in the remaining set? (3.)

Ask another child to find the numeral card for the number of members in the remaining set and place it one the flannel board at the right of the other cards. Review the meaning of the numerals and their placement. That is, the first numeral represents the number of the original set; the second, the number of the subset removed; and the third, the number of the remaining set.

Draw a chart with three columns on the chalkboard and provide each child with set materials. Ask the children to make a set of 6 members. Ask where this numeral should be written in the chart. (In the first column.) Record the numeral and tell the children to remove a subset of two. Ask where this numeral should be written in the chart. After recording the numeral in the second column, determine the number of members in the remaining set. Record the numeral in the third column.

Repeat this process with several other sets. Various children may be given an opportunity to record the numerals.

USING THE PUPIL'S BOOK, page 128:

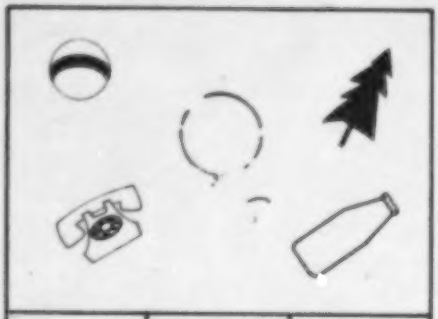
Since the number of the subset to be removed is not indicated in any way, the teacher should work with the children to determine the number of members in the set, then name the number of the subset to be removed (any number less than the number of the original set), and ask for the numbers of members in the remaining set. It is important that the children name the number in each situation and record the numeral in the proper space.

Children may indicate the subset to be removed by either ringing the subset or making an X on each member of the subset.

Page 129:

In each box, the number of members in the subset to be removed is given. The children are to record in the space provided the number of members in the original set and then ring or mark out the subset to be removed. After recording this numeral, record the number of members in the remaining set.

Remove a subset



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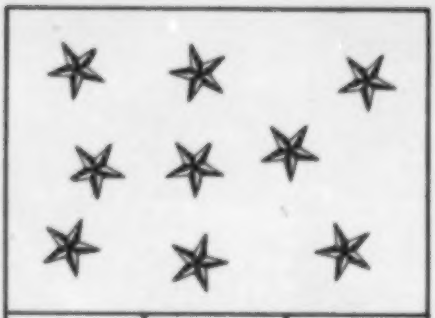

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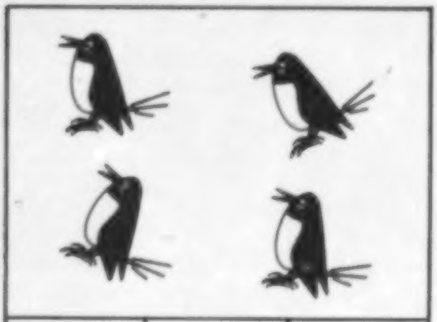

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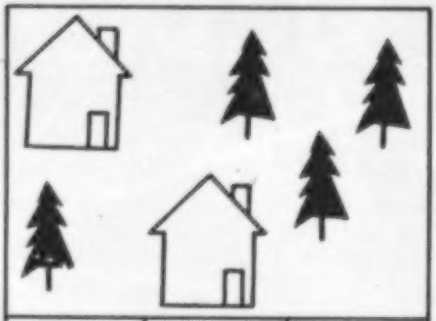

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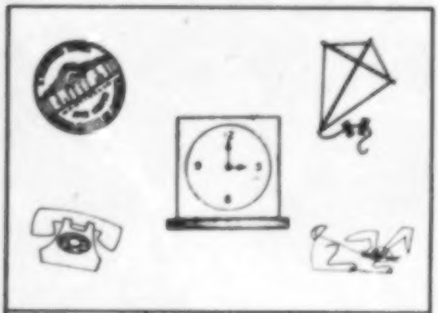

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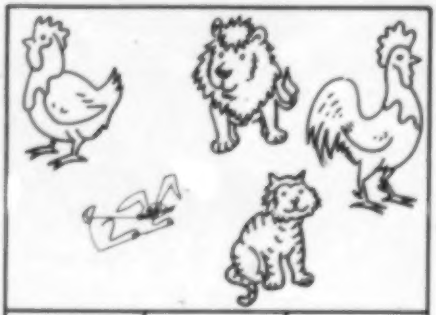

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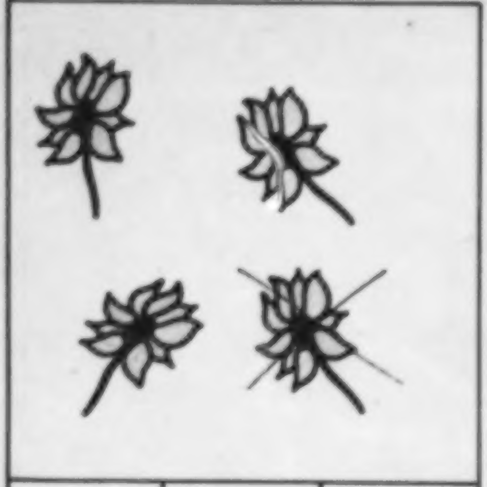
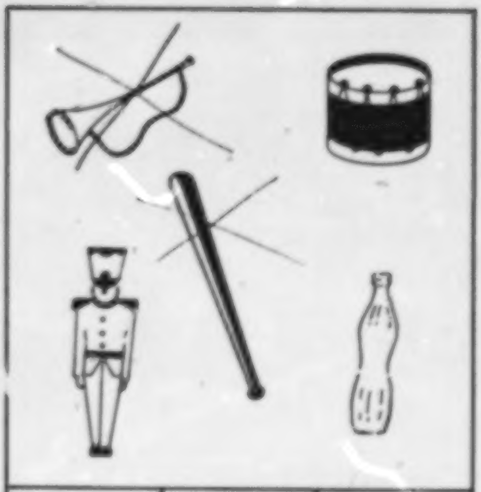
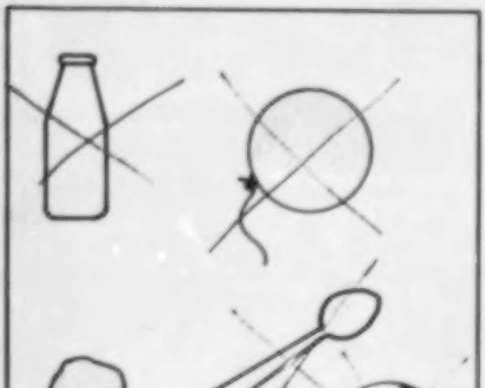
Remobing subsets



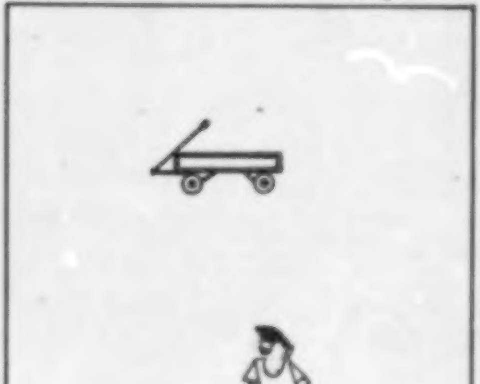
4	1	3



5	2	3



5	4	1



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- Repeat the procedure suggested at the beginning of this section. Then continue.

> The numerals at the bottom of the flannel board can be used to make an equation.
> We are subtracting the number 2 from 5.
> The result is the number 3.
> We say 5 minus 2 equals 3.
> We write the equation, 5 - 2 = 3.

As a review you might ask children the following questions:

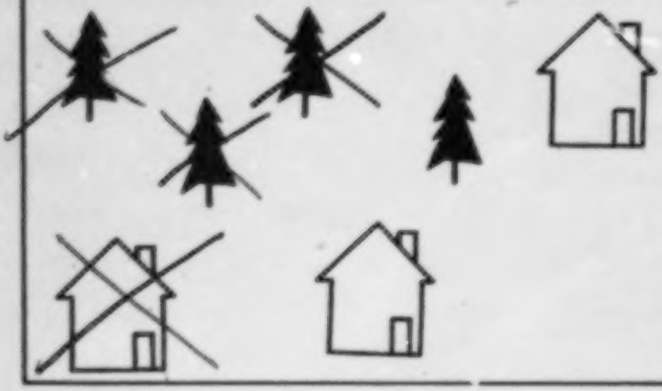
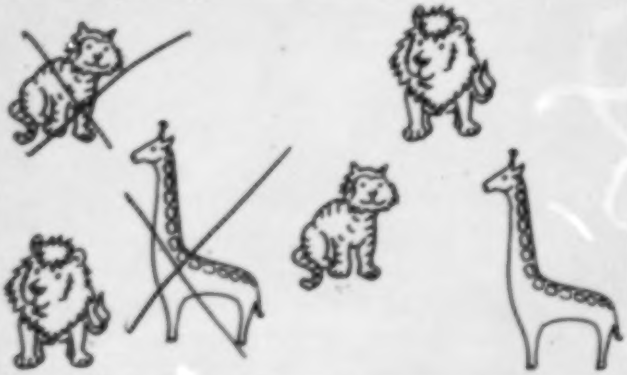
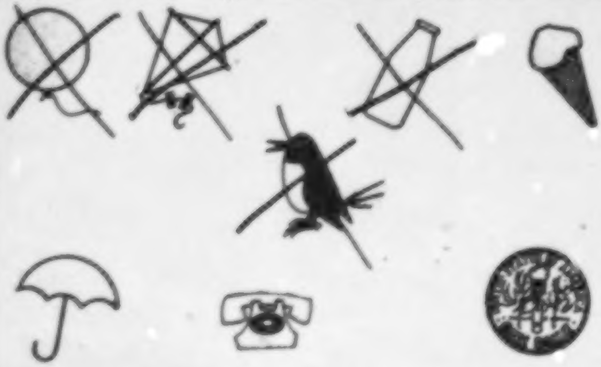
> Which set has 5 members? (The set we started with.)
> Which set has 2 members? (The set we removed.)
> Which set has 3 members? (The remaining set.)
> What is the equation? (5 minus 2 equals 3.)

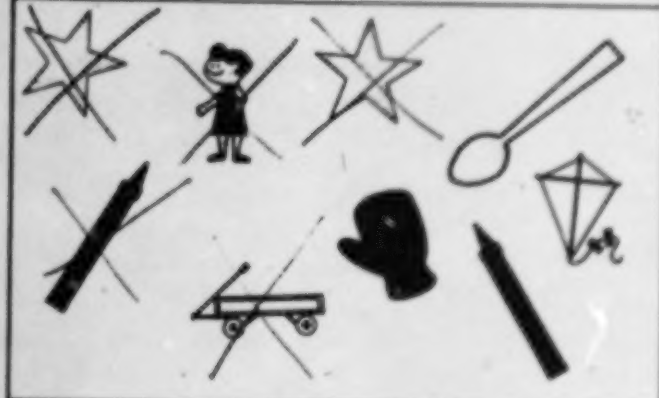
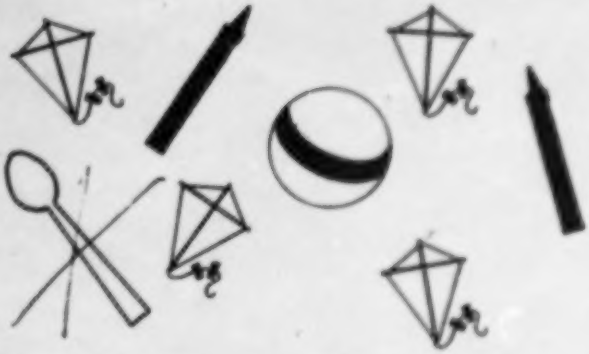
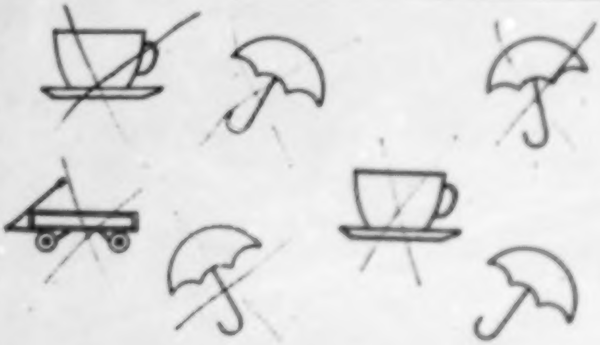
In further development of this lesson, the numerals should be recorded on a chart similar to that used previously. The first column is for the number of members in the starting set; the second column is for the number of members in the set that was removed; and the third column is for the number of members in the remaining set. Write the equation which corresponds to each experience. The completed chart might look like this:

Set	Set removed	Remaining set	Equation
5	2	3	5 - 2 = 3
4	1	3	4 - 1 = 3
2	2	0	2 - 2 = 0
3	1	2	3 - 1 = 2

USING THE PUPIL'S BOOK, pages 130-131:

Direct the children to identify the number of members of the set. They are to indicate the subset to be removed by ringing or marking out the members of the subset. They are to write the numerals which would be used to make a record. Then ask the children to complete the equations.

Remove a subset	Equation
	$\boxed{7}\boxed{4}\boxed{3}$ $\boxed{7}-\boxed{4}=\boxed{3}$
	$\boxed{6}\boxed{2}\boxed{4}$ $\boxed{6}-\boxed{2}=\boxed{4}$
	$\boxed{8}\boxed{4}\boxed{4}$ $\boxed{8}-\boxed{4}=\boxed{4}$

Remove a subset	Equation
	9 | 5 | 4 9 - 5 = 4
	8 | 1 | 7 8 - 1 = 7
	7 | 6 | 1 7 - 6 = 1

USING THE PUPIL'S BOOK, page 132:

Provide each child with set materials. Work with them as they use these materials to complete the equations, e.g., display a set of 5 and remove a subset of 1 to determine that the remaining set has 4 members. Do not permit children to attempt to complete equations unless you are positive they can manipulate the materials correctly.

These pages are suggestive of the type of worksheet that will bridge the gap between the manipulation of set materials and the abstract work on the text page. No doubt you will want to make other worksheets of a similar nature at any one of these stages. Do not underestimate the value of using concrete materials as an aid in completing the work on the page. The child who can use the materials correctly even though he cannot yet work at the abstract level has a tremendous advantage over the child who cannot. The former has a strategy for completing the page, the latter hasn't even that.

In addition, the use of concrete materials enables the teacher to observe if the child understands abstractions. For example, the child who sees the equation $6 - 2 = \square$ and displays a set of 6, removes a set of 2, and writes 4 in the frame, obviously understands this definition of subtraction. In contrast, a child who does not understand and has not differentiated between subtraction and addition might very well display a set of 6 and then a set of 2 with the result that he writes 8 in the frame.

It is not likely in the first introduction to subtraction that many children will be able to do the abstract thinking without the set objects. Do not worry that children need these materials. Insist that they use them until you are confident that the need no longer exists.

Write the equations

$5 - 1 = \boxed{4}$

$1 - 1 = \boxed{0}$

$3 - 1 = \boxed{2}$

$4 - 3 = \boxed{1}$

$6 - 2 = \boxed{4}$

$3 - 2 = \boxed{1}$

$4 - 2 = \boxed{2}$

$2 - 1 = \boxed{1}$

$5 - 2 = \boxed{3}$

$7 - 5 = \boxed{2}$

FURTHER ACTIVITIES:

1. Distribute sets of small objects. Display the numeral 7. Ask each child to "show" a set with that many members on his desk. Display the numeral 4. Ask children to remove a subset with that many members from the sets on their desk.

   How many members are in the remaining set? (3.)

   Display the numeral 3.

   What is an equation that describes removing a subset with 4 members from a set with 7 members? (7 minus 4 equals 3.)

   Write the equation on the blackboard. Emphasize the placement of the minus and equals symbols.

   Continue with these procedures using other numerals. Various children may be asked to write the equation on the chalkboard. This enables the teacher to note those children who are not able to place the symbols correctly.

2. To implement work with removing a set, have the children imagine a birthday cake with 6 to 9 candles. For illustrative purposes 8 will be used in the discussion here. Place numeral cards 0 through 8 in a box. (cards 0 - 6 if 6 candles, etc.) Draw a card from the box and show it to the children. Tell them this is the number of candles which you were able to blow out with one try.

   Identify the number of candles which would still be burning. (If necessary, a row of candles with detachable flames might be placed on the flannel board. When the number indicates how many are blown out is shown, that number of flames could be removed.) Each experience should be followed by writing an equation.

A chart can be made for these experiences to show the different equations which were formed. Then when a child draws a card which has been drawn earlier he can be asked to find the equation which represents this. Only the different equations would be added to the chart.

• SOLVING PROBLEMS

Use sets of objects and dramatization in order to solve these problems.

Five rabbits were in my garden.
Two rabbits hopped away.
Then how many rabbits were in my garden?
What is it that we want to find out? (How many rabbits are in the garden after some go away.)
Yes, the question is, "HOW MANY RABBITS WERE LEFT (remained) IN MY GARDEN after some hopped away?"
Describe the first set you heard about when I read the story. (A set of 5 rabbits in a garden.)

Have a child use materials on flannel board to show a set that matches the set of five rabbits.

What happened to this set of rabbits? (Two rabbits hopped away.)

Ask a child to show what happened using the objects on the flannel board. Discuss the set of three members that remains after the subset with 2 members is removed.

Do you know the answer to the problem?
What is it? (Some children may answer, "Three.")

Ask if the answer is 3 _____, naming whatever materials were used to represent the rabbits. Bring out again that we are answering the question, "How many rabbits remained in the garden?" We use these materials to help us find the answer, but the answer is, "There are still three rabbits in my garden."

What equation can be written about this story? (5 - 2 = 3.)

Other problems to be developed in the same way:

1. Seven birds were sitting in a tree.
   Four of these birds flew away.
   How many birds were still sitting in the tree?

2. Mark had 5 cents.
   He gave 4 cents to Father.
   Then how many cents did Mark have?
3. Mother baked eight cookies.
   She gave three of them to Susan.
   How many did Mother have then?
4. Judy had 9 pencils.
   She lost 2 of these pencils.
   Then how many pencils does Judy have?